

36. Meteorológiai Tudományos Napok,  
HAS/MTA

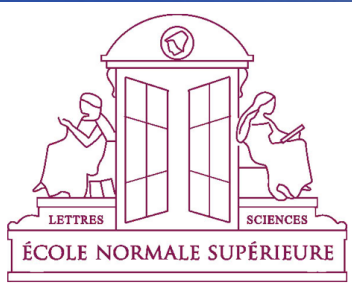


Budapest, 18 November 2010

# Climate Dynamics as a Dynamical Systems Problem

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*Joint work with many people, most recently:*

*R. Berk, U. Penn.; B. Deremble and F. D'Andrea, ENS, Paris;*

*D. Kondrashov, UCLA; and S. Kravtsov, U. Wisconsin, Milwaukee*

*Pls. see this site for further details: <http://www.atmos.ucla.edu/tcd/>,*

# Motivation

- The atmosphere is an **open system** subject to **multiple instabilities** that interact nonlinearly and are limited in energy.
- *Bounded energy and prevalence of dissipation* suggest the existence of **lower-dimensional attractors**; instabilities and observations suggest that these are **strange or worse**.
- Boundedness in phase space and observations also suggest **recurrence of large-scale features** on time scales of interest.
- Two types of recurrent, but unstable features — **fixed points** (“particles”) and **limit cycles** (“waves”) — seem to dominate **low-frequency variability (LFV)**.
- They lie at the basis of two approaches to **long-range forecasting (LRF)**: **Markov chains** and **spectral methods**.
- Simple, “toy” models can provide useful ideas, while the **hierarchical modeling** approach allows one to go back-and-forth between **toy** (“conceptual”) and **detailed** (“realistic”) **models**, and between **models** and **data**.

# Outline

- Motivation and model hierarchy
- Coarse-graining
  - clustering → multiple regimes
  - transition probabilities → predictors
- Empirical model reduction (EMR)
  - models (QG3)
  - observational data (NH flows)
- Conclusions, open questions and bibliography

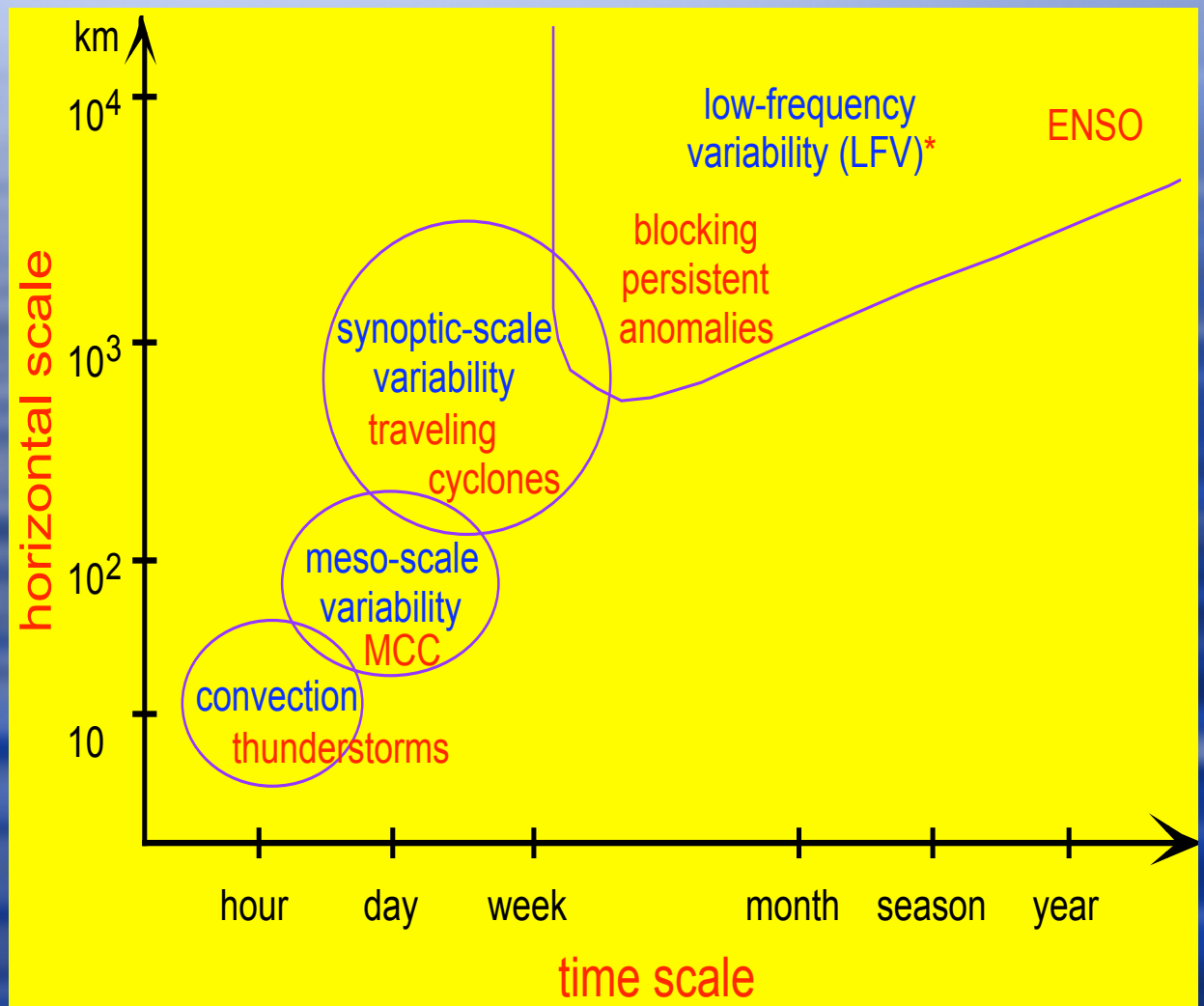


# LFV\* Observations:

## *Multiple space and time scales*

- A **high-variability ridge** lies close to the **diagonal** of the plot (cf. also Fraedrich & Böttger, 1978, JAS)

\* **LFV**  $\cong$  10–100 days (intraseasonal)





# Climate models (atmospheric & coupled) : A classification

- **Temporal**

- stationary, (quasi-)equilibrium
- transient, climate variability

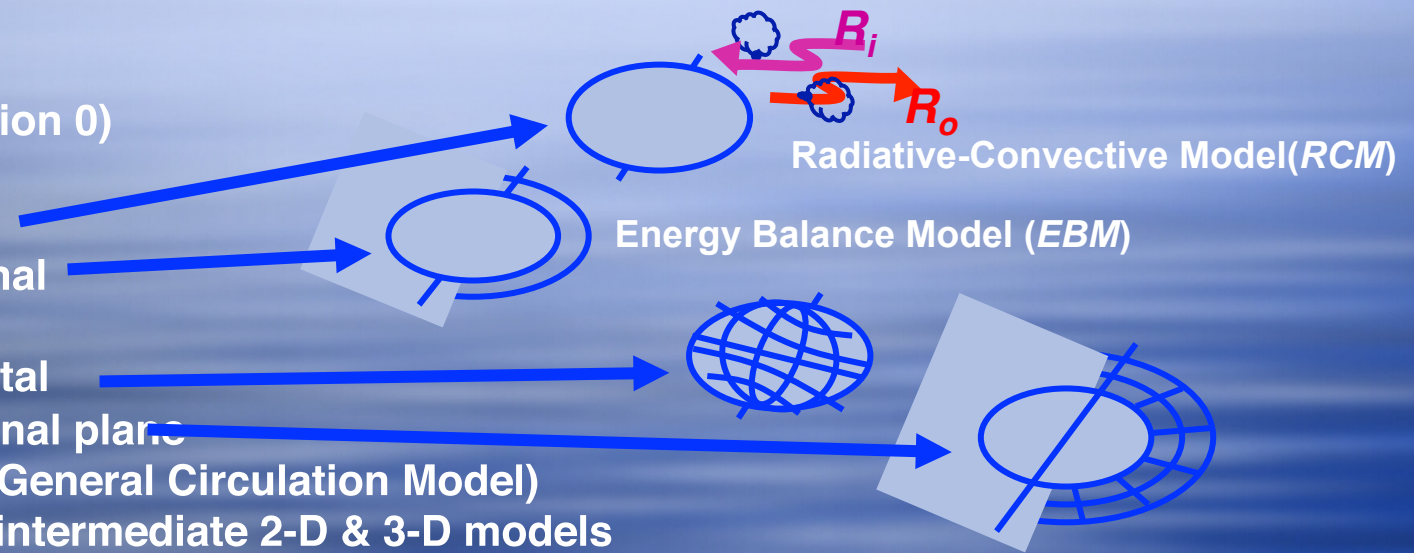
- **Space**

- 0-D (dimension 0)
- 1-D
  - vertical
  - latitudinal
- 2-D
  - horizontal
  - meridional plane
- 3-D, GCMs (General Circulation Model)
- Simple and intermediate 2-D & 3-D models

- **Coupling**

- Partial
  - unidirectional
  - asynchronous, hybrid
- Full

**Hierarchy:** from the simplest to the most elaborate,  
iterative comparison with the observational data



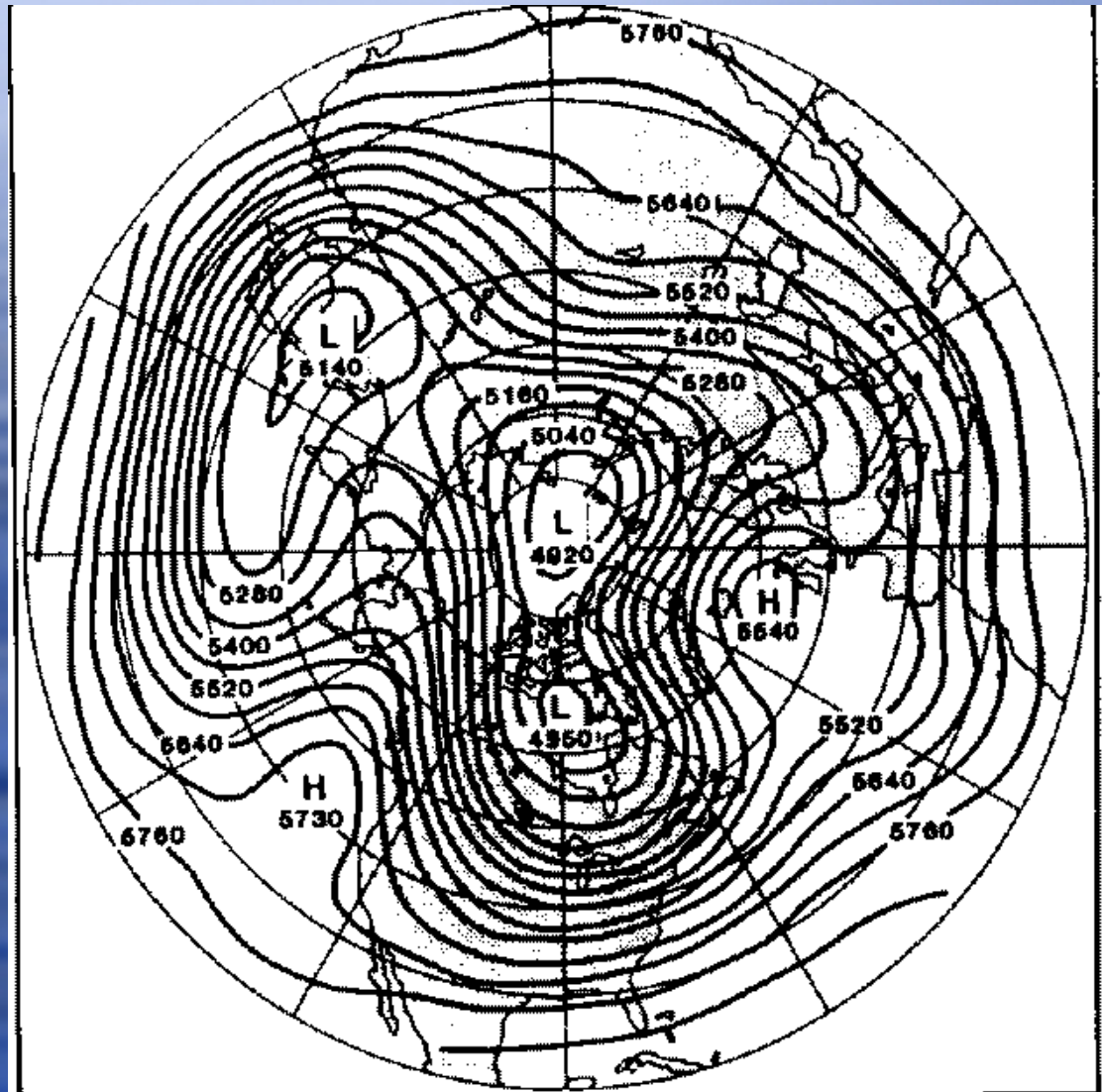
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# Blocking: a paradigm of persistent anomaly

Bauer, Namias, Rex and many others noticed the recurrence and persistence of blocking. J. Charney decided to go beyond “talking about it,” and actually “do something about it.”

Monthly mean 500-hPa map for January 1963 (from Ghil & Childress, 1987)





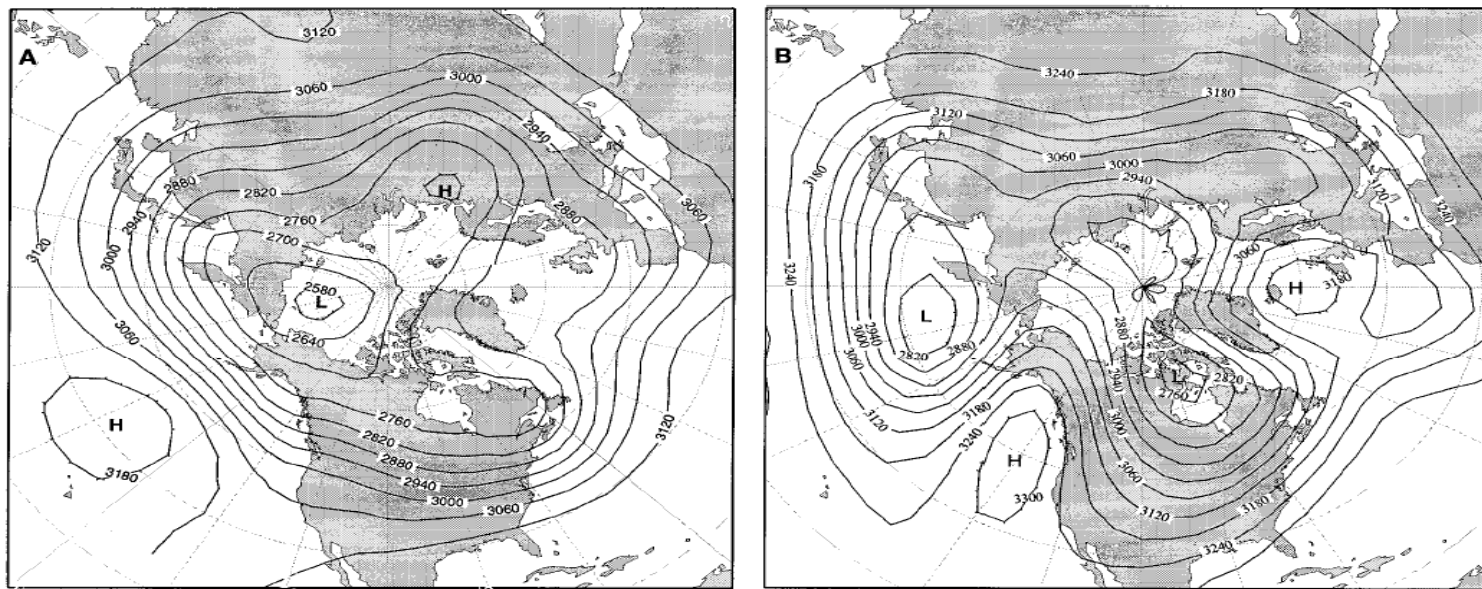
# Transitions Between Blocked and Zonal Flows in a Rotating Annulus with Topography

**Zonal Flow**

13–22 Dec. 1978

**Blocked Flow**

10–19 Jan. 1963



**Fig. 1.** Atmospheric pictures of (A) zonal and (B) blocked flow, showing contour plots of the height (m) of the 700-hPa (700 mbar) surface, with a contour interval of 60 m for both panels. The plots were obtained by averaging 10 days of twice-daily data for (A) 13 to 22 December 1978 and (B) 10 to 19 January 1963; the data are from the National Oceanic and Atmospheric

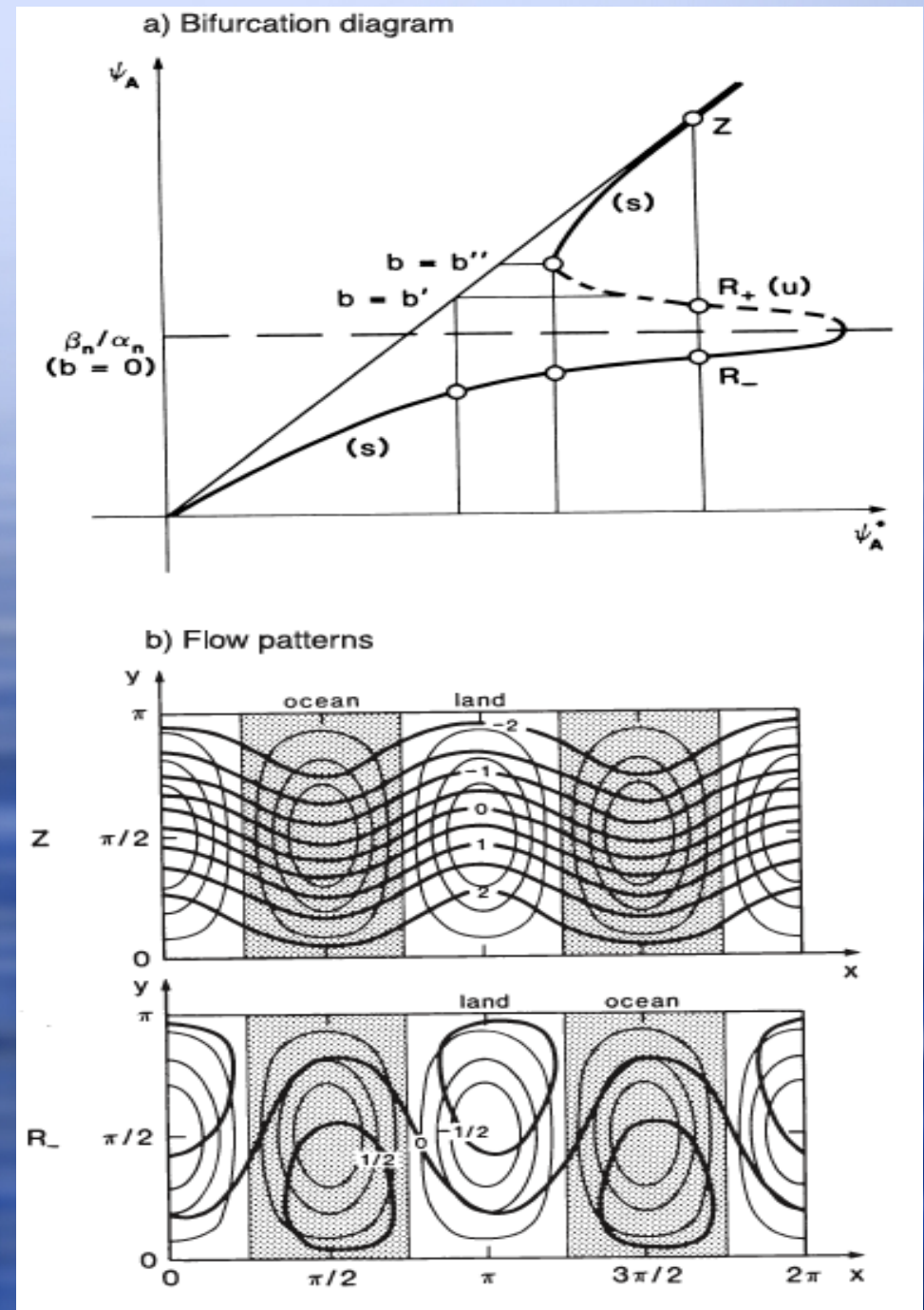
Administration's Climate Analysis Center. The nearly zonal flow of (A) includes quasi-stationary, small-amplitude waves (32). Blocked flow advects cold Arctic air southward over eastern North America or Europe, while decreasing precipitation in the continent's western part (26).

E.R. Weeks, Y. Tian, J. S. Urbach, K. Ide, H. L. Swinney, & M. Ghil,  
1997: *Science*, **278**, 1598–1601.

# A toy model for blocking vs. zonal flow

- Quasi-geostrophic flow in a mid-latitude  $\beta$ -channel, with 3-mode truncation (zonal + 1 wave).
- Topographic resonance leads to multiple equilibria: zonal + blocked.
- Much criticized as “unrealistic.”

Charney & DeVore, 1979:  
*J. Atmos. Sci.*, 36, 1205–1216.



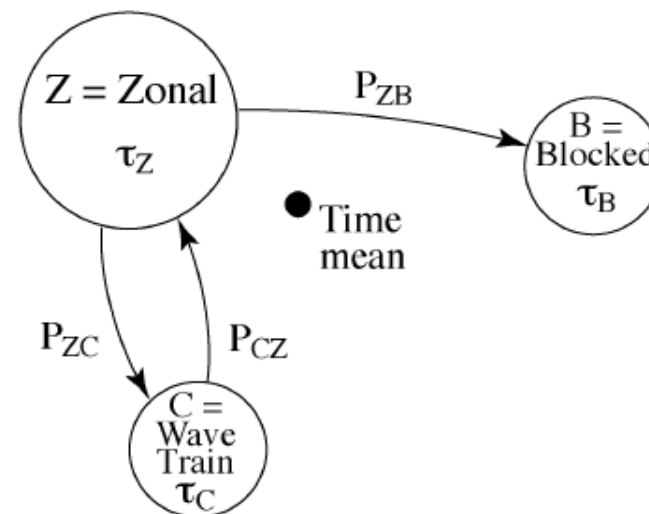


# From Regimes to Markov Chains

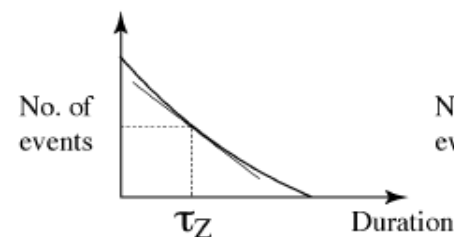
- ◆ Each regime  $R$  has an expected duration  $\tau_R$ .
- ◆ Expected transition probability from regime  $A$  to  $B$  is  $p_{AB}$ .
- ◆ Transitions do NOT occur via the mean state, which is a statistical “accident” or, maybe, the root of the “bifurcation tree.”

From Ghil (1987), in  
Nicolis & Nicolis (eds.).

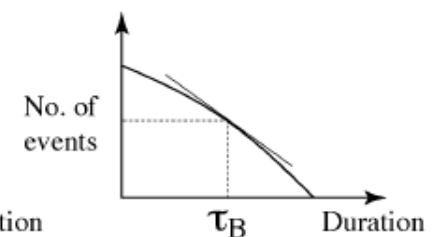
## *Markov-chain description of LFV*



(a)

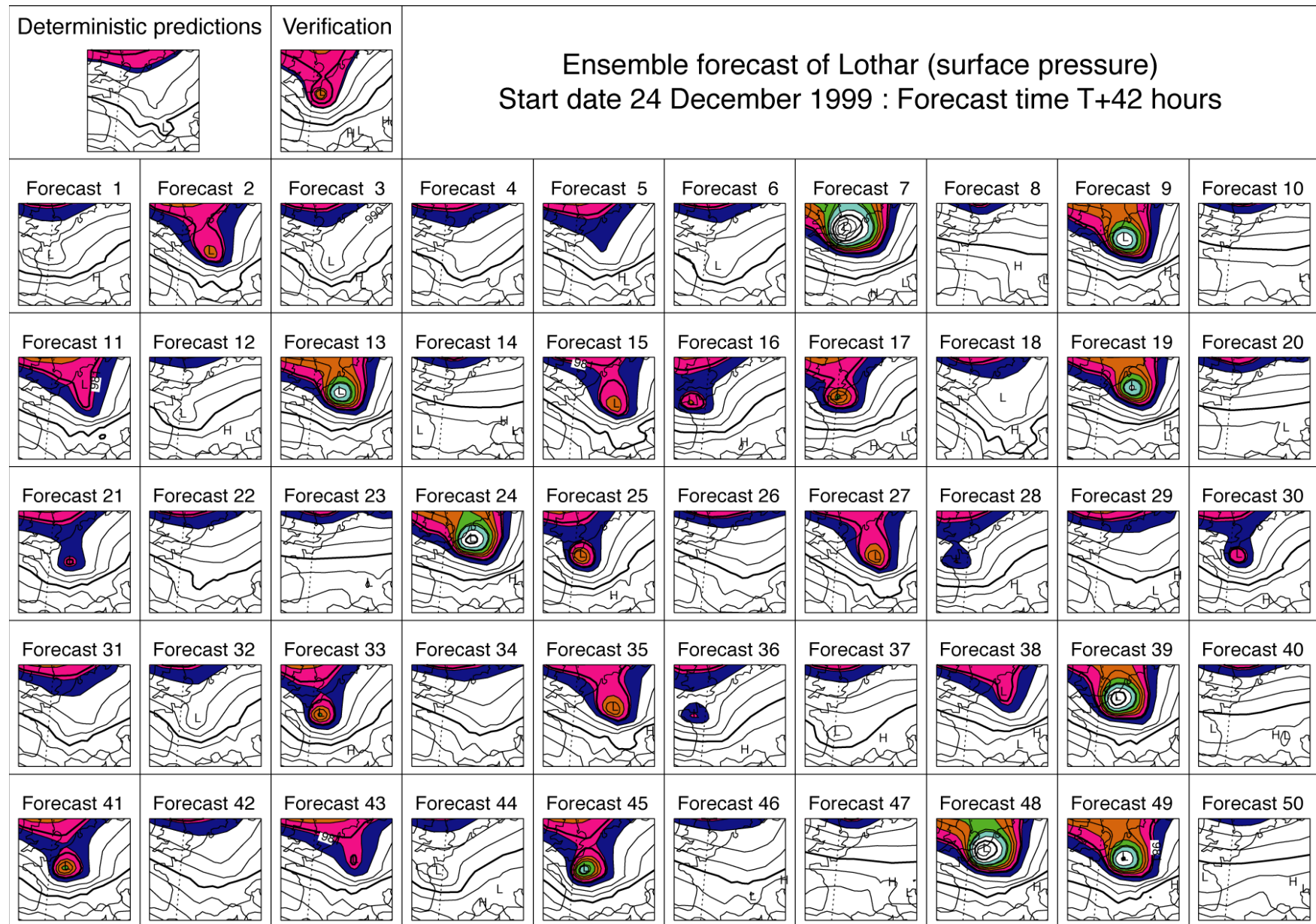


(b)



(c)

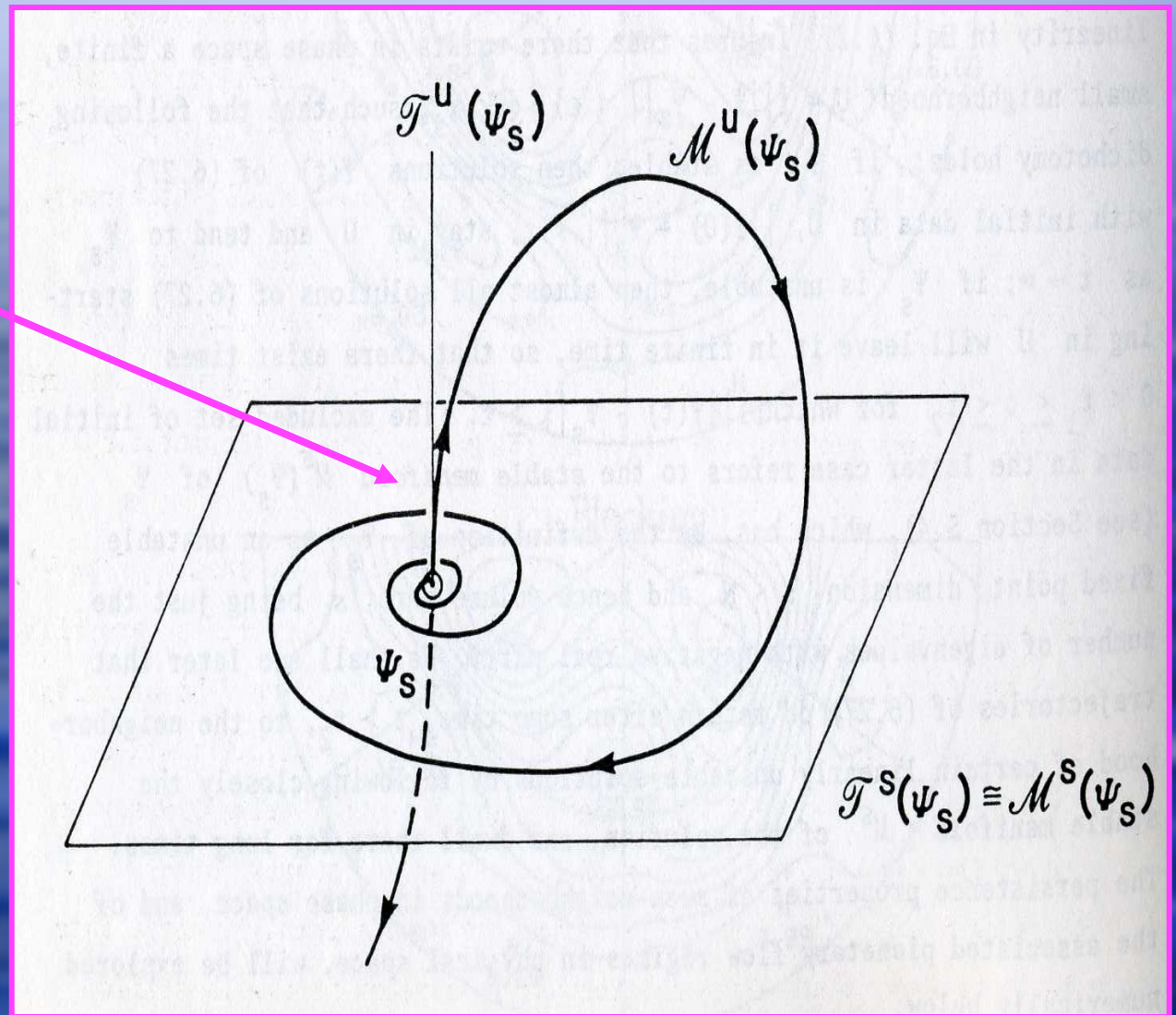




Courtesy Tim Palmer, 2009

# How to get from a regime to another? – I

- ♦ Stochastic perturbations
- ♦ Heteroclinic and homoclinic orbits
- ♦ Chaotic itinerancy
- ♦ All of the above



Ghil & Childress,  
1987: Ch. 6



# Multiple Flow Regimes

## A. Classification schemes

### 1) *By position*

- (i) **Cluster analysis**– **categorical** – NH, Mo & Ghil (1988, *JGR*) – fuzzy
  - NH + sectorial, Michelangeli *et al.* (1995, *JAS*) – hard (*K*-means)
  - **hierarchical** – NH + sectorial, Cheng & Wallace (1993, *JAS*)
- (ii) **PDF estimation** – **univariate**:
  - NH, Benzi *et al.* (1986, *QJRM*S); Hansen & Sutera (1995, *JAS*)
  - **multivariate**:
    - NH, Molteni *et al.* (1990, *QJRM*S); Kimoto & Ghil (1993a, *JAS*)
    - sectorial, Kimoto & Ghil (1993b, *JAS*); Smyth *et al.* (1999, *JAS*)

### 2) *By persistence*

- (iii) **Pattern correlations**
  - NH, Horel (1985, *MWR*); SH, Mo & Ghil (1987, *JAS*)
- (iv) **Minima of tendencies**
  - Models: Legras & Ghil (1985, *JAS*); Mukougawa (1988, *JAS*); Vautard & Legras (1988, *JAS*)
  - Atl.- Eur. sector : Vautard (1990, *MWR*)

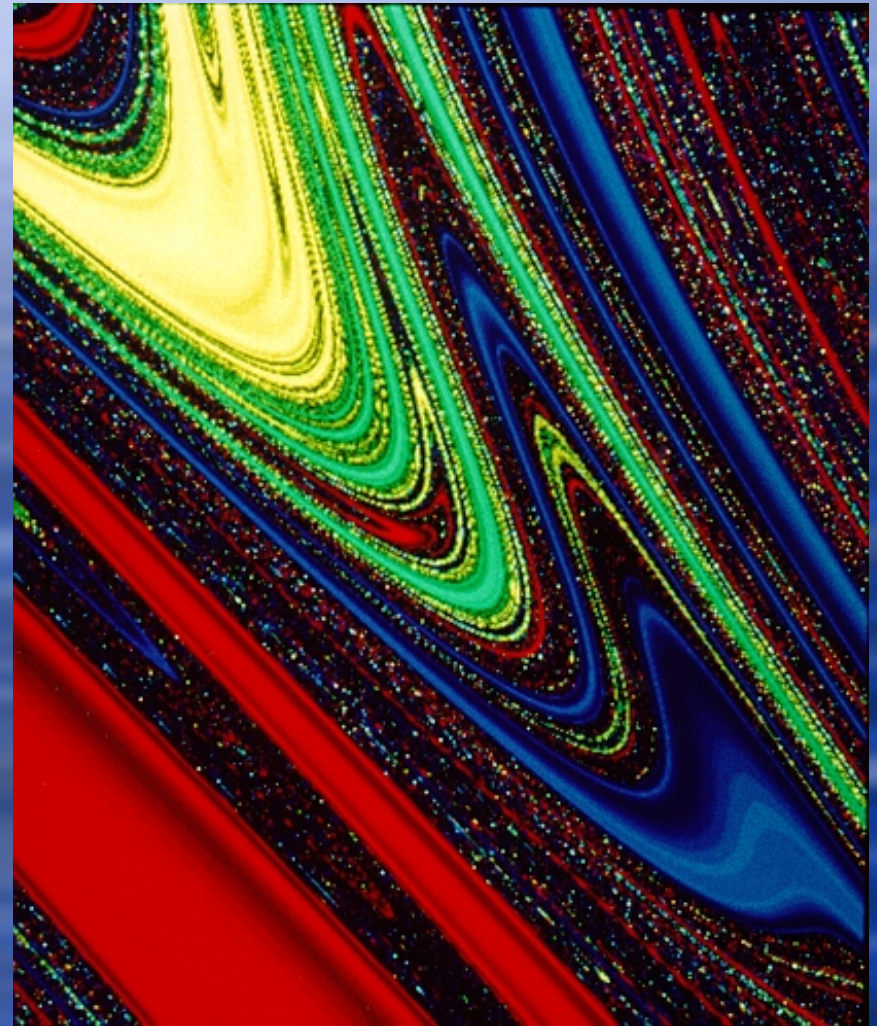
## B. Transition probabilities

- (v) Model & NH – counts (Mo & Ghil, 1988, *JGR*)
- (vi) NH & SH – Monte Carlo (Vautard *et al.*, 1990, *JAS*)



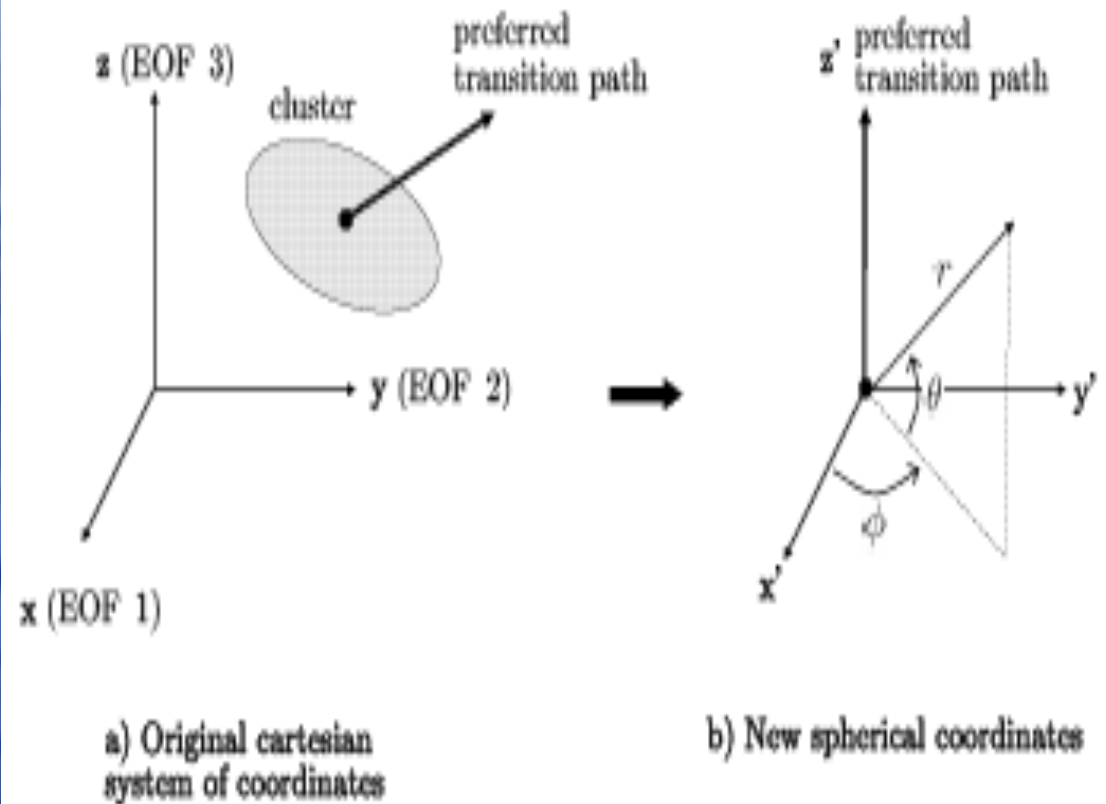
# How to get from a regime to another? – II

- ◆ Even something as simple as a periodically forced damped pendulum can have complex behavior.
- ◆ Here are 4 attractor basins, each with a different type of behavior.
- ◆ Time to get there is shown by brightness of color.



# Preferred Transition Paths

- ◆ Conjectured by Legras & Ghil (*JAS*, 1985) in toy model ( $25 Y_m^n$ ).
- ◆ Captured by Kondrashov *et al.* (*JAS*, 2004) in intermediate QG3 (Marshall & Molteni, 1993) model.
- ◆ Exit angles used as predictors in statistical, random-forests algorithm:
  - for QG3 model by Deloncle *et al.* (*JAS*, 2006);
  - for NH reanalysis data by Kondrashov *et al.* (*Clim. Dyn.*, 2007).



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# Key ideas

- Nonlinear dynamics:  $\dot{\mathbf{x}} = \mathbf{L}\mathbf{x} + \mathbf{N}(\mathbf{x}).$

- Discretized, quadratic:

$$dx_i = (\mathbf{x}^T \mathbf{A}_i \mathbf{x} + \mathbf{b}_i^{(0)} \mathbf{x} + c_i^{(0)}) dt + dr_i^{(0)}; \quad 1 \leq i \leq I.$$

- Multi-level modeling of red noise:

$$dx_i = (\mathbf{x}^T \mathbf{A}_i \mathbf{x} + \mathbf{b}_i^{(0)} \mathbf{x} + c_i^{(0)}) dt + r_i^{(0)} dt,$$

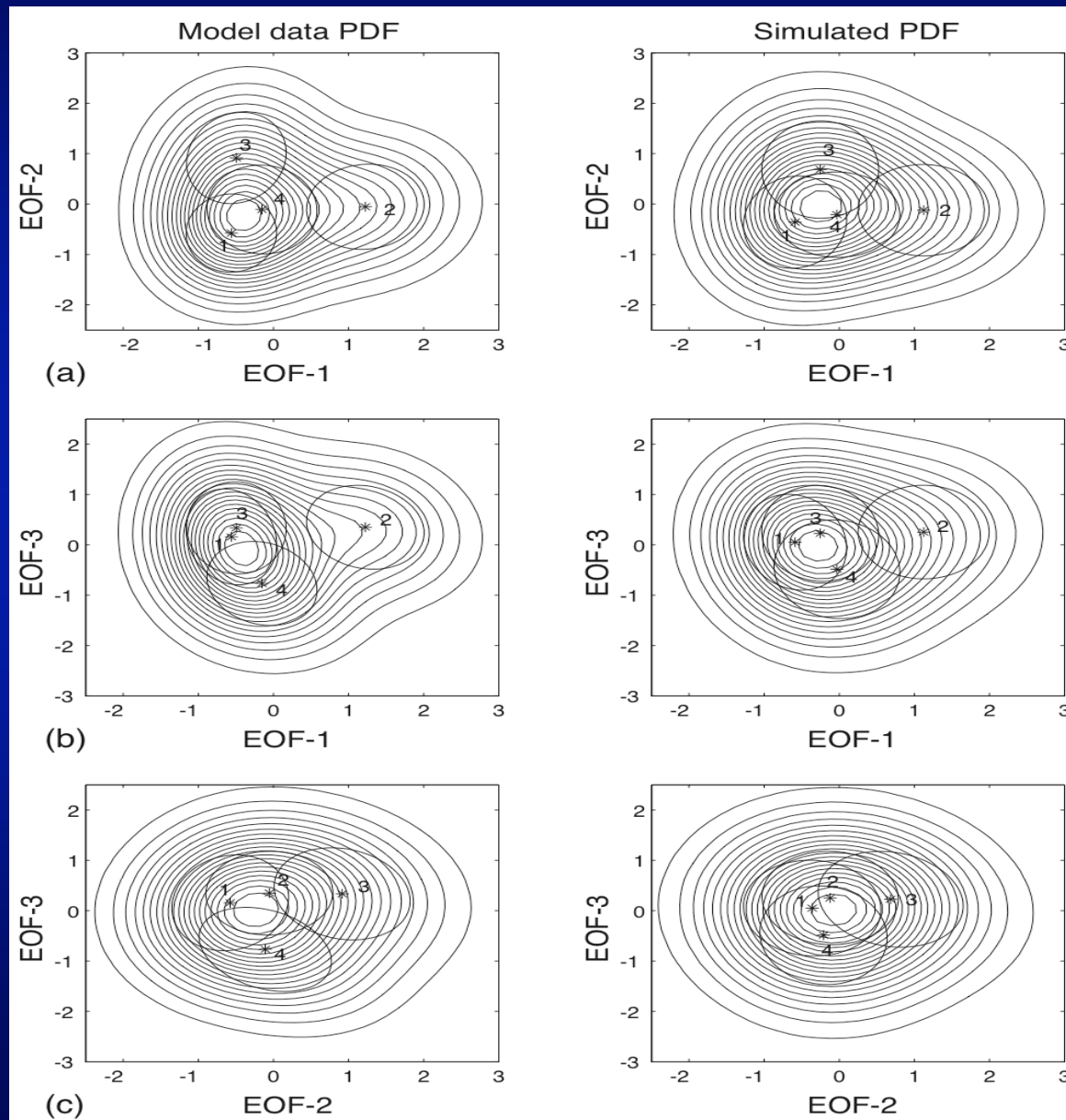
$$dr_i^{(0)} = \mathbf{b}_i^{(1)}[\mathbf{x}, \mathbf{r}^{(0)}] dt + r_i^{(1)} dt,$$

$$dr_i^{(1)} = \mathbf{b}_i^{(2)}[\mathbf{x}, \mathbf{r}^{(0)}, \mathbf{r}^{(1)}] dt + r_i^{(2)} dt,$$

...

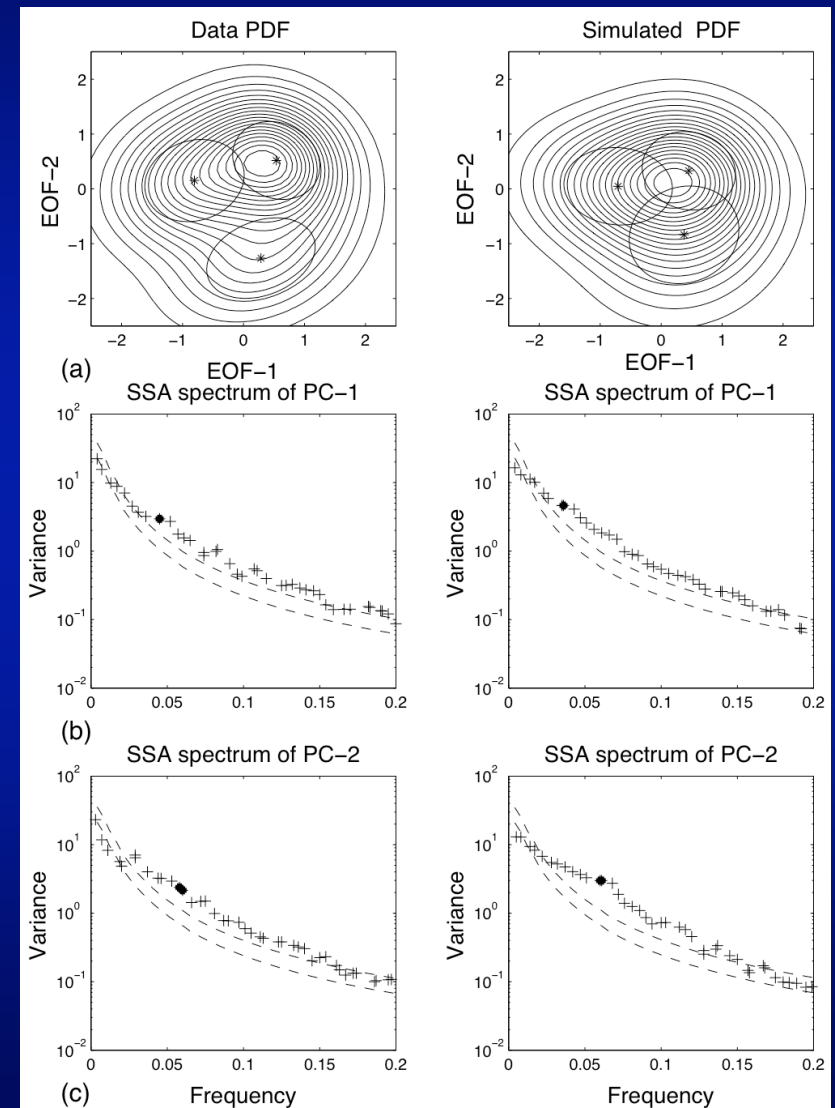
$$dr_i^{(L)} = \mathbf{b}_i^{(L)}[\mathbf{x}, \mathbf{r}^{(0)}, \mathbf{r}^{(1)}, \dots, \mathbf{r}^{(L)}] dt + dr_i^{(L+1)}; \quad 1 \leq i \leq I.$$

# NH LFV in QG3 Model – III



# NH LFV – Observed Heights

- 44 years of daily 700-mb-height winter data
- 12-variable, 2-level model works OK, but dynamical operator has unstable directions: “sanity checks” required.





# NWP Model Performance on Blocking

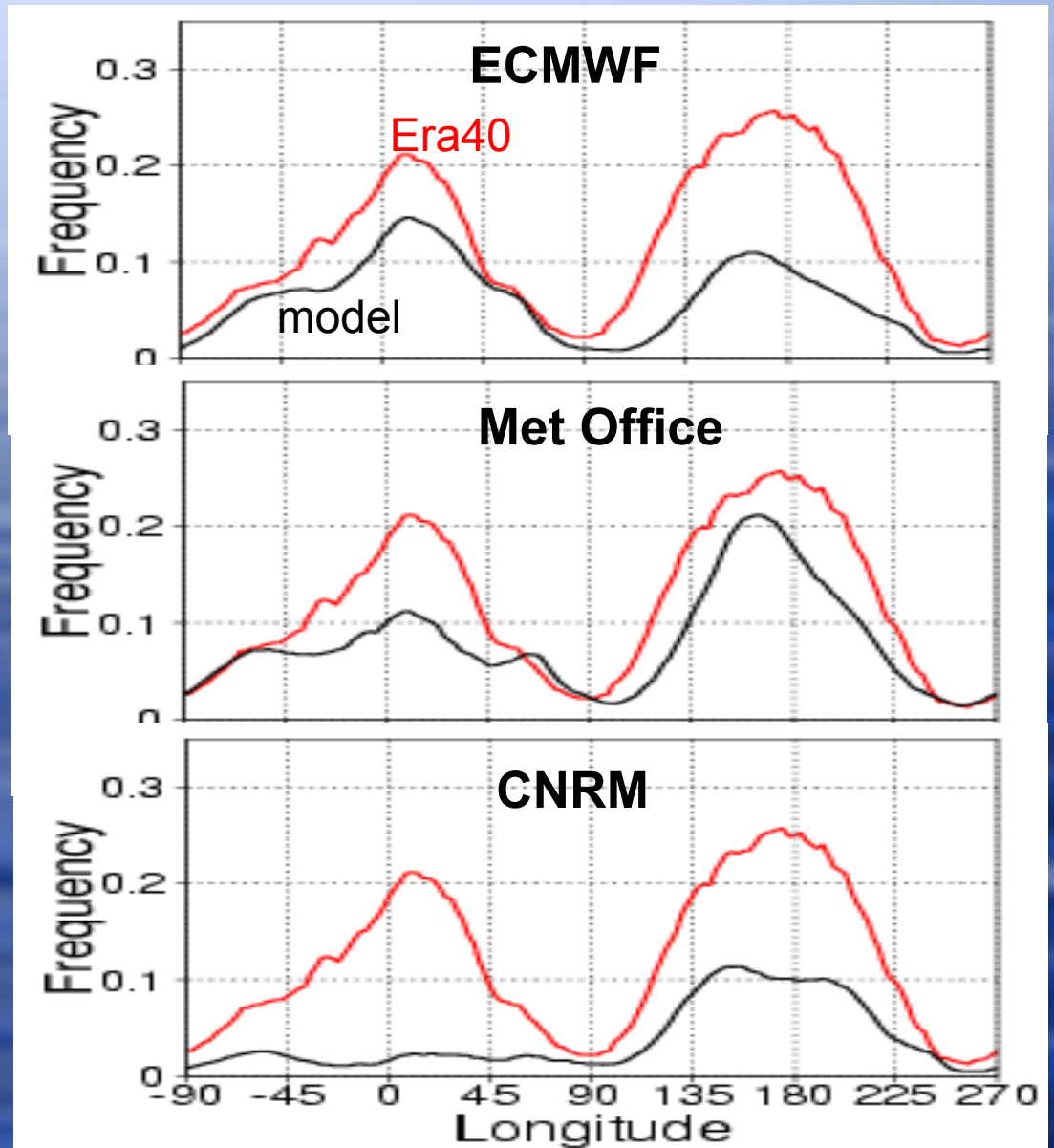
Leading numerical weather prediction (NWP) models still underestimate badly blocking occurrence and persistence:

ECMWF - European Centre

Met Office - United Kingdom

CNRM - Météo-France

T. N. Palmer *et al.* (2008,  
*Bull. Amer. Met. Soc.*)



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# Concluding remarks

- ♦ ***Dynamical systems theory*** provides major insights into nonlinear planetary flows:
  - ♦ **bifurcation theory** helps understand how *spatio-temporal patterns* arise in observed flows and detailed numerical simulations (GCMs);
  - ♦ **ergodic theory** helps connects the *statistics* of the flows (EOFs, PDFs, Markov chains) to their dynamics.
- These theories are most easily understood for **highly simplified models** but they do apply to the *full PDE systems* that govern the actual flows.
- The simplification often consists in a **reduced number of modes** (d-o-f's), as well as simplified *physical processes*.
- **Increase gradually model resolution and sophistication:** move **up & down model hierarchy**, continuously **comparing models with the data**.

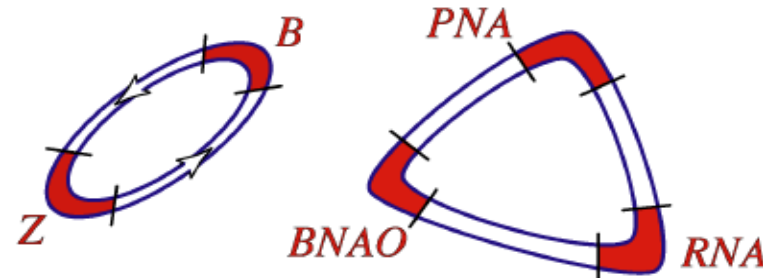
# A few questions left:

- Are the **regimes** but slow phases of the **oscillations**?
- Are the **oscillations** but instabilities of particular **fixed points**?
- How about both?
  - chaotic itinerancy
- How about neither?
  - just interference of linear waves;
  - just red noise.

## "Waves vs. Particles"

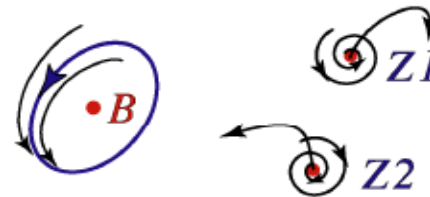
### in Atmospheric Low-Frequency Variability

1. Are the **regimes** but slow phases of the **oscillations**?



Kimoto & Ghil  
(JAS, 1993a, b)

2. Are the **oscillations** but instabilities of particular **equilibria**?

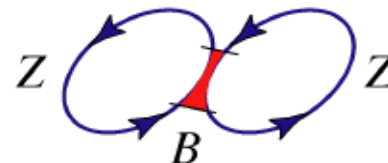


Legras & Ghil  
(JAS, 1985)

3. How about **both**: "chaotic itinerancy" (Itoh & Kimoto, JAS, 1999)

4. How about **neither**? Null hypotheses:

- a) It's all due to **interference of linear waves**, *e.g.*, neutrally stable Rossby waves;



Lindzen *et al.*  
(JAS, 1982)

- b) It's all due to **red noise** — Hasselmann (*Tellus*, 1976), Mitchell (*Quatern. Res.*, 1976), Penland & co. (Magorian, Sardeshmukh, 1990s).



# Some general references

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