Some theoretical questions and development of MASH for homogenization of Standard Deviation

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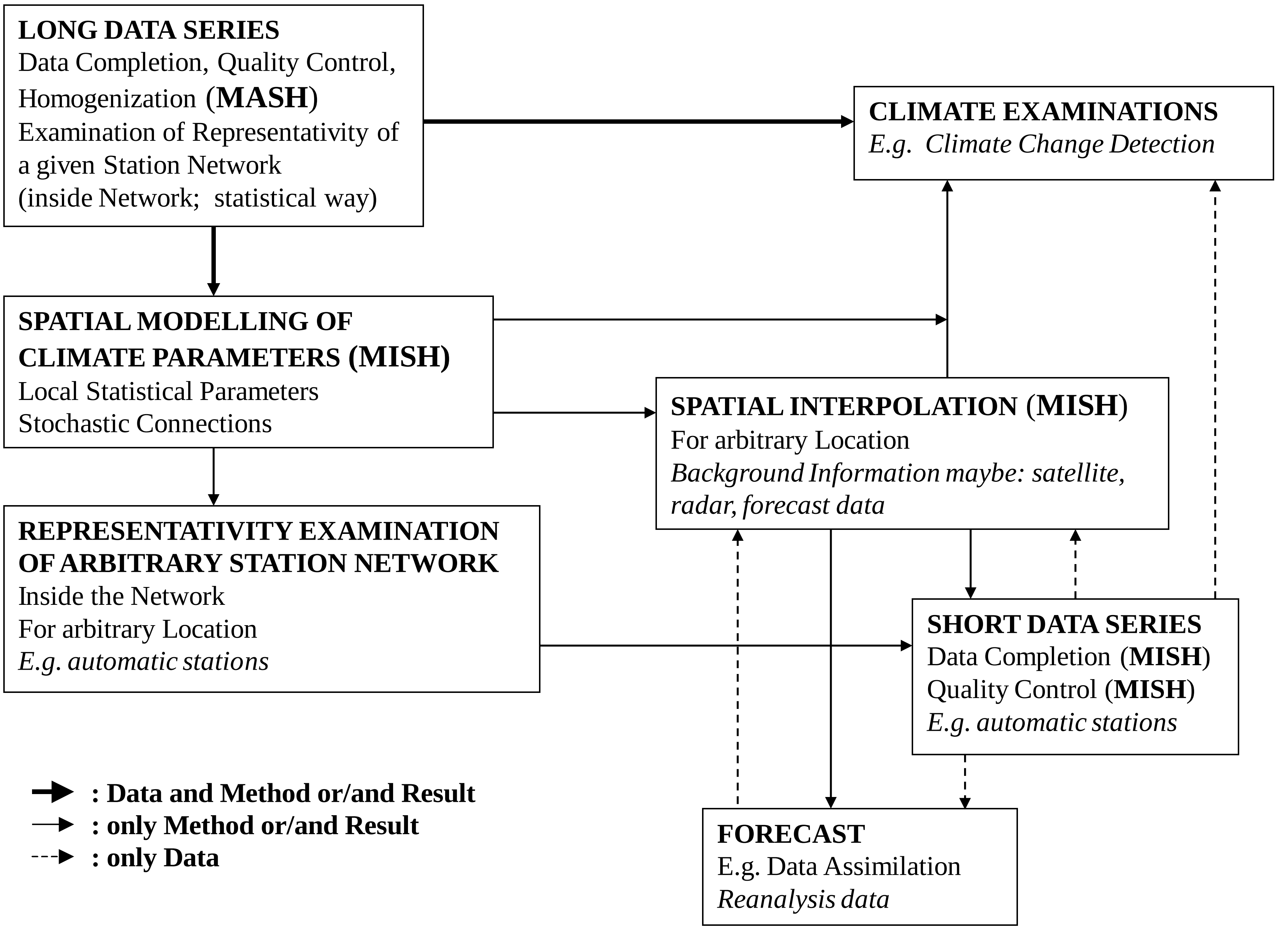
Possible Connection of Topics and Systems



MATHEMATICAL FORMULATION OF HOMOGENIZATION

Let us assume we have daily or monthly data series.

$Y_1(t)\ \left(t=1,2,..,n\right)$: candidate series of the new observing system

$Y_2(t)\ \left(t=1,2,..,n\right)$: candidate series of the old observing system

$1 \leq T < n$: change-point

Before T: series $Y_2(t)\ \left(t=1,2,..,T\right)$ can be used

After T: series $Y_1(t)\ \left(t=T+1,..,n\right)$ can be used

Theoretical cumulative distribution functions (CDF):

$$F_{1,t}(y)=\mathrm{P}\left(Y_1(t)<y\right)\ ,\quad F_{2,t}(y)=\mathrm{P}\left(Y_2(t)<y\right)\ ,\qquad t=1,2,..,n$$

Functions $F_{1,t}(y)$, $F_{2,t}(y)$ change in time (e.g. climate change)!

Theoretical formulation of homogenization

Inhomogeneity: $F_{2,t}(y) \neq F_{1,t}(y) \quad (t = 1,2,..,T)$

Homogenization of $Y_2(t) \quad (t = 1,2,..,T)$:

$Y_{1,2h}(t) = F_{1,t}^{-1}\left(F_{2,t}\left(Y_2(t)\right)\right)$, then $\mathrm{P}\left(Y_{1,2h}(t) < y\right) = F_{1,t}(y)$

Transfer function: $F_{1,t}^{-1}\left(F_{2,t}\left(y\right)\right)$, Quantile function: $F_{1,t}^{-1}(p)$

Remark

The basis of the Quantile Matching methods can be integrated into the general theory. However these methods developed in practice mainly for daily data are very weak empiric methods.

It is not real mathematics! (good heuristics with poor mathematics)

The correction formula: $Y_{1,2h}(t) = F_{1,t}^{-1}\left(F_{2,t}\left(Y_2(t)\right)\right) \quad \left(t = 1,2,..,T\right)$

Problems

Estimation, detection of change point(s) T ?

Estimation of distribution functions $F_{1,t}(y)$, $F_{2,t}(y)$ $\left(t = 1,2,..,T\right)$?

i, $F_{1,t}(y)$, $F_{2,t}(y)$ change in time (annual cycle, climate change)

ii, No sample for $F_{1,t}(y)$ $\left(t = 1,2,..,T\right)$

The problem is insolvable in general case!

Only relative methods can be used with some assumptions.

Statistically speaking, some assumptions have to be made!

Special but basic case: Normal Distribution (e.g. temperature)

Theorem.

Let us assume normal distribution,

$$Y_1(t) \in N\big(E_1(t), D_1(t)\big)\ , \quad Y_2(t) \in N\big(E_2(t), D_2(t)\big) \quad \big(t = 1,2,..,n\big)$$

$E_1(t), E_2(t)$: means $\quad D_1(t), D_2(t)$: standard deviations

Then the transfer function of homogenization:

$$Y_{1,2h}(t) = F_{1,t}^{-1}\Big(F_{2,t}\big(Y_2(t)\big)\Big) = E_1(t) + \frac{D_1(t)}{D_2(t)}\big(Y_2(t) - E_2(t)\big) \quad \big(t = 1,2,..,T\big)$$

Remarks:

i, A simple linear function and there is no “tail distribution” problem!

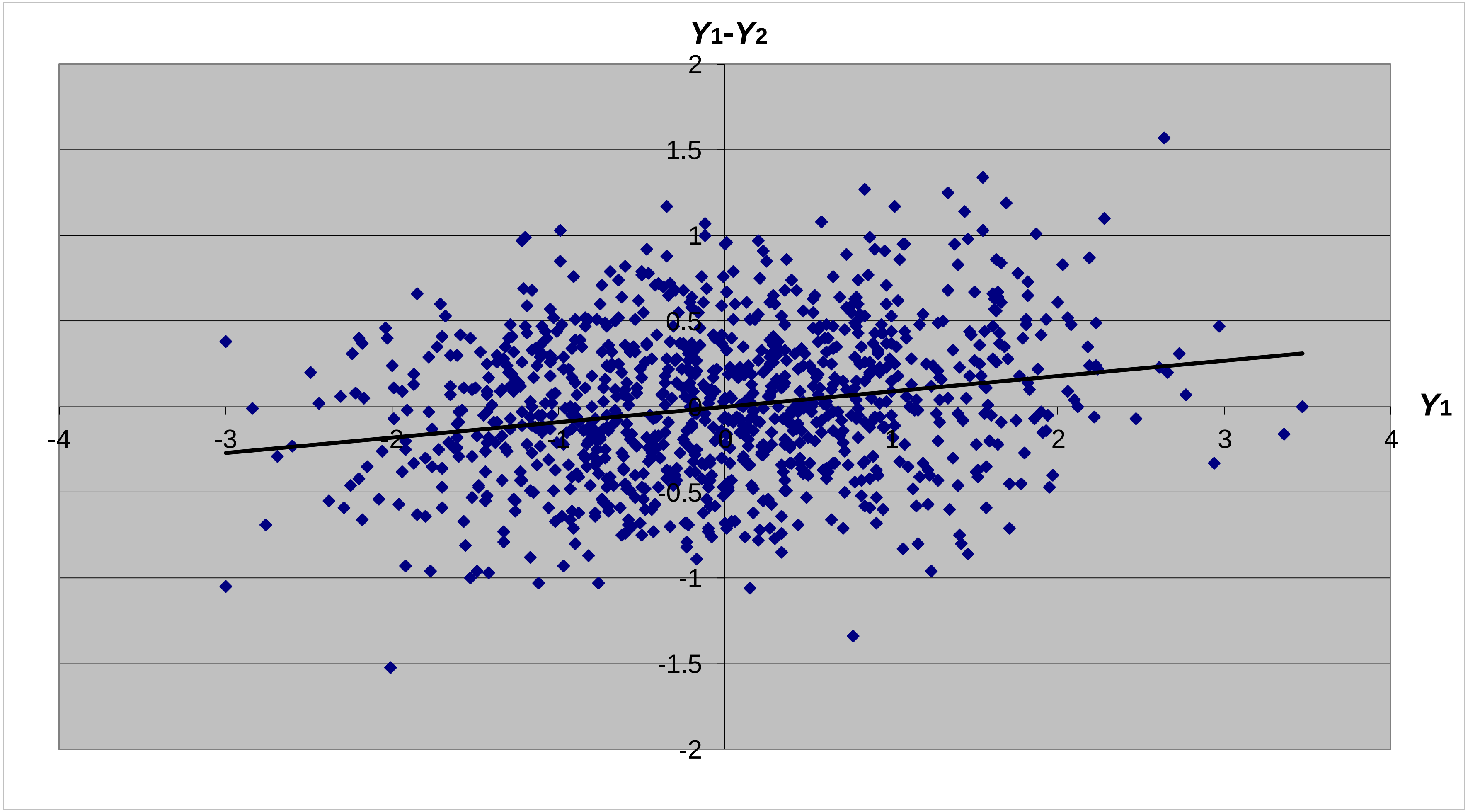
ii, Only the mean (***E***) and standard deviation (***D***) must be homogenized!

Parallel measurements, "tail distribution" problem or rather a natural phenomenon?

Example by Monte-Carlo method for the natural dependence of $Y_1 - Y_2$ on Y_1

Generated series: $Y_1(t) \in N(0,1),\ Y_2(t) \in N(0,1),\ \mathrm{corr}(Y_1(t), Y_2(t)) = \rho = 0.9 \quad (t = 1,..,1000)$

Difference series: $Y_1(t) - Y_2(t)$, $\quad \mathrm{E}(Y_1(t) - Y_2(t) \mid Y_1(t)) = (1-\rho) \cdot Y_1(t) = 0.1 \cdot Y_1(t)$



Problems

Estimation of $E_1(t)\,, D_1(t)\,, E_2(t)\,, D_2(t)\ \left(t = 1,2,..,T\right)$

- change in time (climate change)
- no sample for $E_1(t)\,, D_1(t)\ \left(t = 1,2,..,T\right)$

Assumptions

a, $D_2(t)/D_1(t) = D_{21}, \quad E_2(t) - E_1(t) = E_{21} \quad \left(t = 1,2,..,T\right)$

b, $D_{21} = 1\ , \quad E_2(t) - E_1(t) = E_{21} \qquad \left(t = 1,2,..,T\right)$

$$\Rightarrow \qquad Y_{1,2h}(t) = Y_2(t) - E_{21} \qquad \left(t = 1,2,..,T\right)\ ,$$

Homogenization in mean applied in practice for monthly series.

What is in the Practice?

A popular procedure

1. Homogenization of monthly mean series:

   Break points detection, correction of mean (*E*)

   Assumption: homogeneity of higher order moments (e.g. st. deviation (*D*))

2. Homogenization of daily series:

   Trial to homogenize also the higher order moments
   (Quantile Matching, Spline)

   Used monthly information: only the detected break points

Contradiction

- Inhomogeneity of higher moments, **daily: yes** versus **monthly: no** ?

  It is not adequate mathematical model for standard deviation (*D*)!

- Why are not used the monthly correction factors for daily homogenization?

Theorem

Daily data: $Y(t)\ \left(t=1,..,30\right)$, monthly mean: $\bar{Y}=\frac{1}{30}\sum_{t=1}^{30}Y(t)$

Monthly variable for examination of standard deviation (D): $S=\sqrt{\frac{1}{29}\sum_{t=2}^{30}\left(Y(t)-Y(t-1)\right)^2}$

Daily data with inhomogeneity in mean (E) and standard deviation (D):

$$Y_{ih}(t)=\alpha\cdot\left(Y(t)-\mathrm{E}\left(Y(t)\right)\right)+\mathrm{E}\left(Y(t)\right)+\beta \qquad \left(t=1,..,30\right)$$

$$\mathrm{E}\left(Y_{ih}(t)\right)=\mathrm{E}\left(Y(t)\right)+\beta\ ,\quad \mathrm{D}\left(Y_{ih}(t)\right)=\alpha\cdot\mathrm{D}\left(Y(t)\right)$$

The appropriate monthly variables: $\bar{Y}_{ih}=\frac{1}{30}\sum_{t=1}^{30}Y_{ih}(t)$, $S_{ih}=\sqrt{\frac{1}{29}\sum_{t=2}^{30}\left(Y_{ih}(t)-Y_{ih}(t-1)\right)^2}$

i, Then the monthly mean is also inhomogeneous in mean (E) and standard deviation (D):

$$\mathrm{E}\left(\bar{Y}_{ih}\right)=\mathrm{E}\left(\bar{Y}\right)+\beta \quad \text{and} \quad \mathrm{D}\left(\bar{Y}_{ih}\right)=\alpha\cdot\mathrm{D}\left(\bar{Y}\right)$$

ii, Moreover variable S_{ih} can be used to estimate the inhomogeneity of standard deviation (D):

$$\mathrm{E}\left(S_{ih}\right)=\alpha\cdot\mathrm{E}\left(S\right)$$

An alternative procedure developed in MASH

1. Homogenization of monthly series $S(t)$, $\overline{Y}(t)$.

Homogenization of series $S(t)$ by multiplicative model.

- Break points detection, estimation of inhomogeneity of st. deviation (D).

Correction of standard deviation of series $\overline{Y}(t)$.

Homogenization of corrected series $\overline{Y}(t)$ by additive model.

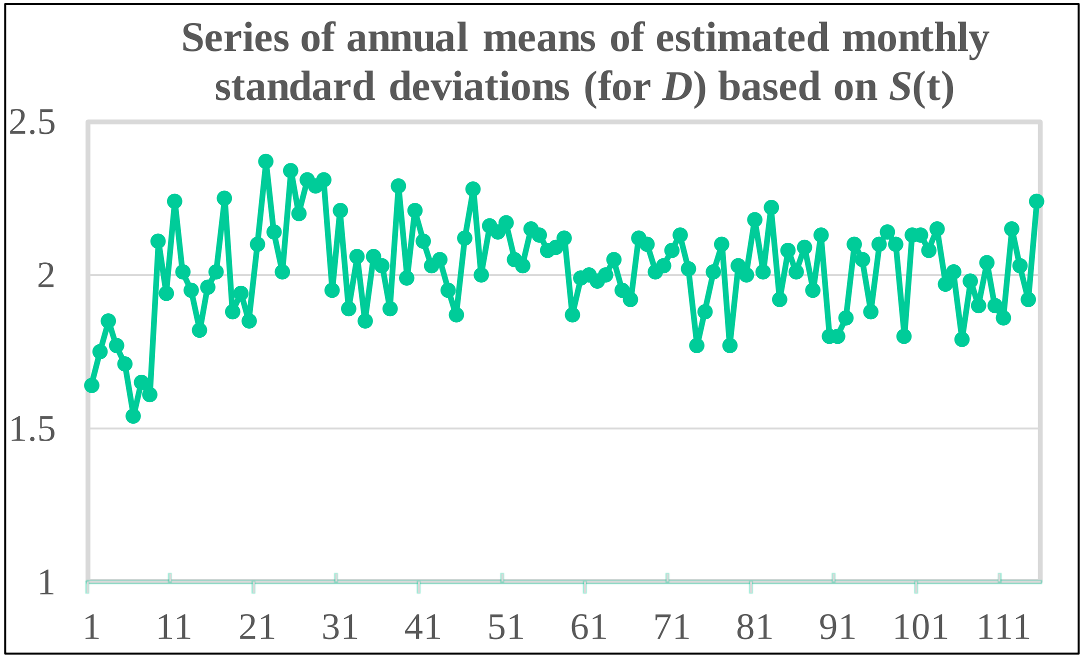
- Break points detection, estimation of the inhomogeneity of mean (E).

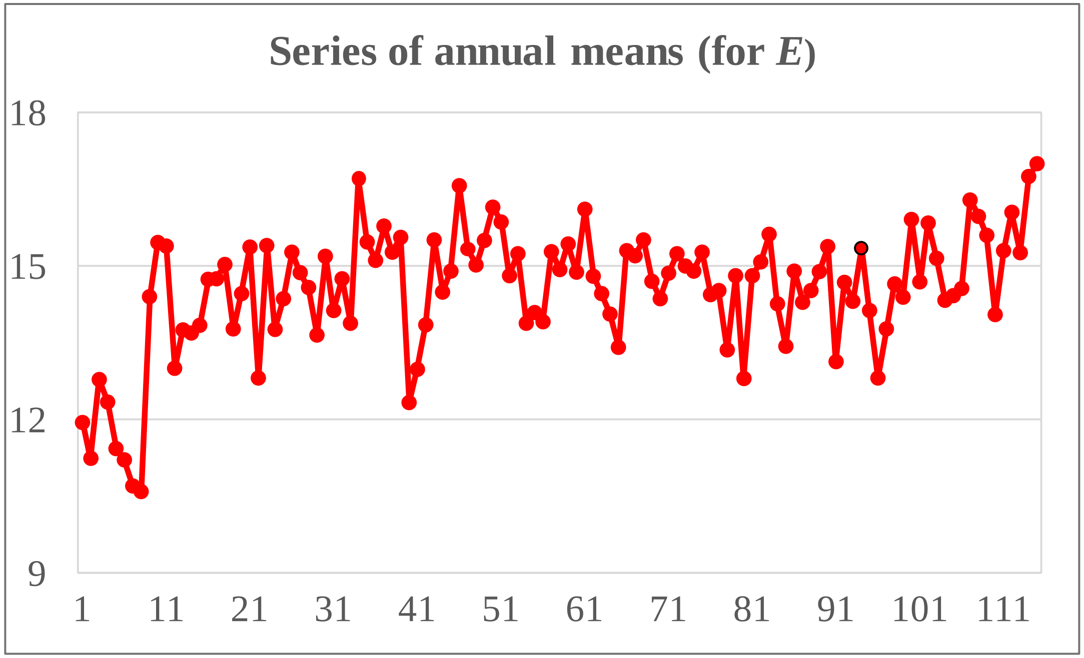
Assumption: homogeneity of higher order (>2) moments.

This assumption is always right in case of normal distribution!

2. Homogenization of daily series

Homogenization of mean and standard deviation on the basis of the monthly results. The used monthly information are the break points and the monthly corrections of the mean (E) and standard deviation (D).





Maximum temperature series of Miskolc (in Hungary) 1901-2015

Inhomogeneity in 1901-1908, measured Réaumur: $\mathrm{Re} = 0.8 \cdot \mathrm{C}$

$$\mathrm{E}\left(Y_{ih}(t)\right) = 0.8 \cdot \mathrm{E}\left(Y(t)\right)$$

$$\mathrm{D}\left(Y_{ih}(t)\right) = 0.8 \cdot \mathrm{D}\left(Y(t)\right)$$

Software MASHv4.01 (Multiple Analysis of Series for Homogenization)
(*T. Szentimrey*) (under development)

The MASH system is based on homogenization of monthly series derived from daily series. The procedures depend on the distribution of climate elements.

Quasi normal distribution (e.g. temperature)

Beside the monthly mean series another type monthly series are also derived. These series are homogenized by multiplicative model for standard deviation (*D*). The monthly mean series corrected in standard deviation are homogenized by additive model for mean (*E*).

Quasi lognormal distribution (e.g. precipitation)

Monthly mean or sum series are homogenized by multiplicative model.

Software MASHv4.01 (Multiple Analysis of Series for Homogenization)
(*T. Szentimrey*)

Homogenization of monthly series:

- Relative homogeneity test procedure.
- Step by step iteration procedure: the role of series (candidate, reference) changes step by step in the course of the procedure.
- Additive or multiplicative model can be used depending on the distribution.
- Providing the homogeneity of the seasonal and annual series as well.
- Metadata (probable dates of break points) can be used automatically.
- The homogenization results and the metadata can be verified.

Homogenization of daily series:

- Based on the detected monthly inhomogeneities.
- Including Quality Control and missing data completion for daily data.

Remark

The aim of MASH is not the full automation and we are sceptic in such an aspect. However our intention is to obtain such a flexible automatic system wherein the mechanic, labour-intensive procedures are automated, moreover the operating process can be controlled simply and the accidental mistakes can be corrected easily. The basic idea of this conception is to control the results via the verification tables generated automatically during the automatic procedures.

15 Hungarian July Mean Temperature Series 1901-2015

Test Statistics for St. Deviation (D) Before Homogenization

Critical value (significance level 0.01): 28.00

Series	TSB	Series	TSB	Series	TSB
7	201.40	8	168.65	13	126.68
9	123.38	4	121.03	14	94.02
12	83.32	2	78.07	5	63.54
6	58.47	11	44.14	15	43.91
10	32.60	1	25.54	3	17.14

AVERAGE: 85.46

Test Statistics for Mean (E) Before Homogenization

Critical value (significance level 0.05): 21.76

Series	TSB	Series	TSB	Series	TSB
12	1674.66	7	388.59	8	237.88
3	230.71	10	224.70	5	211.41
6	188.81	11	154.68	14	125.35
4	82.50	9	72.61	15	57.57
1	53.55	13	49.91	2	32.95

AVERAGE: 252.39

15 Hungarian July Mean Temperature Series 1901-2015

Test Statistics for St. Deviation (D) After Homogenization

Critical value (significance level 0.01): 28.00

Series	TSA	Series	TSA	Series	TSA
13	36.93	14	32.50	4	32.29
8	26.56	12	25.73	7	23.87
9	23.71	5	23.37	2	22.18
1	19.85	3	19.70	11	18.49
6	16.55	10	16.55	15	14.82

AVERAGE: 23.54

Test Statistics for Mean (E) After Homogenization

Critical value (significance level 0.05): 21.76

Series	TSA	Series	TSA	Series	TSA
5	25.55	3	23.24	13	21.64
14	21.19	7	19.43	9	18.53
6	18.02	15	16.98	8	16.61
12	16.49	11	16.25	4	15.70
2	14.29	10	13.28	1	11.69

AVERAGE: 17.93

15 Hungarian July Mean Temperature Series 1901-2015

Estimated Inhomogeneities for St. Deviation (D)(%)

Series	IHD	Series	IHD	Series	IHD
8	8.05	9	7.98	4	6.73
12	4.88	7	4.08	11	3.59
6	3.33	2	2.43	15	2.22
5	2.16	13	2.02	10	1.70
1	1.57	14	1.34	3	0.54

AVERAGE: 3.51

$$D_{ih}(t) = D(t) \cdot IHD(t) \quad (t = 1,.,n), \qquad IHD = \frac{100}{n}\sum_{t=1}^{n}\left|IHD(t) - 1\right|$$

Estimated Inhomogeneities for Mean (E) (°C)

Series	IHE	Series	IHE	Series	IHE
3	0.80	8	0.55	15	0.53
7	0.52	12	0.48	10	0.48
14	0.31	6	0.31	5	0.29
11	0.24	1	0.23	4	0.14
9	0.13	2	0.09	13	0.08

AVERAGE: 0.35

$$E_{ih}(t) = E(t) + IHE(t) \quad (t = 1,.,n), \qquad IHE = \frac{1}{n}\sum_{t=1}^{n}\left|IHE(t)\right|$$

There is no royal road!

Thank you for your attention!