Interreg
Danube Transnational Programme
DriDanube

Methodology of Drought Risk assessment in DriDanube project

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DriDanube – Drought Risk in the Danube Region
Project co-funded by European Union funds (ERDF, IPA)

Outline

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3 Algorithm for drought risk assessment

4 Software RED for drought risk assessment

5 Risk Maps

6 Software Practice (tomorrow)

Introduction

One of the aim of the DriDanube project is the drought risk assessment and the main responsible partner of this activity is the Hungarian Meteorological Service.

However, unfortunately there is no commonly accepted procedure for this purpose therefore it was necessary to review and prepare the mathematical methodology that can be applied for drought risk assessment in this project. VARIMAX is a subcontractor for the mathematical and software development.

The recommended approach was as it is described in European Commission’s Risk Assessment and Mapping Guidelines for Disaster Management.

EUROPEAN COMMISSION

Risk Assessment and Mapping Guidelines for Disaster Management

5. Risk Assessment Methods

5.1. Conceptual Framework and Basic Methodology

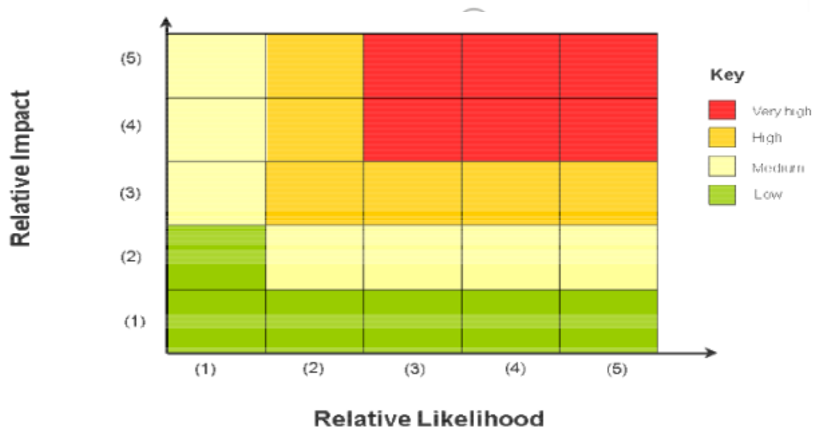
According to ISO 31010, risks are the combination of the consequences of an event or hazard and the associated likelihood of its occurrence.

Risk = hazard impact * probability of occurrence

Qualitative Formulation only!

What is the Risk Matrix? It is a tool for Qualitative Risk Mapping!

Schema of Qualitative Risk Mapping according to the Methodology of SEERISK



Qualitative Probability Map → **Use Risk Matrix to assign a risk level** → **Qualitative Risk Map**

Qualitative Impact Map → **Use Risk Matrix to assign a risk level**

The two qualitative maps (probability and impact) can be overlaid. Each pixel will be given a colour which will indicate a risk level according to the risk matrix.

Qualitative data will lead to the production of a risk matrix that will then be used as a base for the production of a qualitative risk map.

But we want to implement Quantitative Risk Mapping!

General Mathematical Methodology

(for meteorological drought risk assessment)

Meteorological variables in space (s) and time (t year):

$$\mathbf{X}(\mathbf{s},t) = \left[X_1(\mathbf{s},t),\ldots.,X_N(\mathbf{s},t)\right]^{\mathrm{T}} \qquad \mathbf{s} \in S \ \text{(e.g. grid)},\ t = 1,..,n$$

i, Continuous distribution, joint density function: $f\left(\mathbf{x};\, \mathbf{s}\right)$

ii, Discrete distribution: $p_i\left(\mathbf{s}\right) = \mathrm{P}\left(\mathbf{X}\left(\mathbf{s},t\right) = \mathbf{a}_i\right),\ i = 1,2,\ldots.$

Meteorological drought loss (hazard impact): $\mathrm{Loss}(\mathbf{s},t)$

Meteorological drought loss (hazard impact) function:

$$\mathrm{E}\left(\mathrm{Loss}(\mathbf{s},t) \middle| \mathbf{X}(\mathbf{s},t)\right) = L\left(\mathbf{X}(\mathbf{s},t)\right)$$

$L(\mathbf{X}(\mathbf{s},t))$ that is the conditional expectation (regression) of the meteorological loss given the meteorological variables.

Meteorological drought risk (depends on location):

$$Risk(\mathbf{s}) = \mathrm{E}\left(\mathrm{Loss}(\mathbf{s},t)\right) = \mathrm{E}\left(L\left(\mathbf{X}(\mathbf{s},t)\right)\right)$$

i.e. the expected value of the meteorological loss function.

The basic cases

i, Continuous distribution: $Risk(\mathbf{s}) = \mathrm{E}\left(L\left(\mathbf{X}(\mathbf{s},t)\right)\right) = \int\limits_{R^N} L(\mathbf{x})\, f(\mathbf{x};\,\mathbf{s})\; d\mathbf{x}$

ii, Discrete distribution: $Risk(\mathbf{s}) = \mathrm{E}\left(L\left(\mathbf{X}(\mathbf{s},t)\right)\right) = \sum\limits_{i=1} L(\mathbf{a}_i)\cdot p_i(\mathbf{s})$

Special case:

If $L(\mathbf{a}_1)$=hazard impact, $p_1(\mathbf{s})$=probability of occurence

$L(\mathbf{a}_2) = 0$, $p_2(\mathbf{s}) = 1 - p_1(\mathbf{s})$, then

$Risk(\mathbf{s}) = \sum\limits_{i=1}^{2} L(\mathbf{a}_i)\cdot p_i(\mathbf{s})$ =**hazard impact*probability of occurence**

Estimation of meteorological drought risk

$$\hat{Risk}(\mathbf{s}) = \frac{1}{n}\sum_{t=1}^{n} \hat{L}\left(\mathbf{X}(\mathbf{s},t)\right)$$

where $\hat{L}(\mathbf{X}(\mathbf{s},t))$ is an estimation of loss function $L(\mathbf{X}(\mathbf{s},t))$,

since $Risk(\mathbf{s}) = \mathrm{E}\left(L\left(\mathbf{X}(\mathbf{s},t)\right)\right)$

BUT: In general there is no sample for the drought loss values!

Loss function derivation from yield function and drought identification

Crop yield values: $\mathrm{Y}(\mathbf{s},t)=\mathrm{Yield}(\mathbf{s},t)$

Meteorological yield function:

$$\mathrm{E}\left(\mathrm{Yield}(\mathbf{s},t)\,\middle|\,\mathbf{X}(\mathbf{s},t)\right)=Y\left(\mathbf{X}(\mathbf{s},t)\right)$$

that is the conditional expectation (regression) of the crop yield given the meteorological variables.

Drought identification by set D: there is drought if $\mathbf{X}(\mathbf{s},t)\in D$

The probability of drought: $P_D=\mathrm{P}\left(\mathbf{X}(\mathbf{s},t)\in D\right)$

Loss function based on yield function and drought identification

$$L(\mathbf{X}(\mathbf{s},t)) = 0 \quad \text{if} \quad \mathbf{X}(\mathbf{s},t) \notin D \text{ (there is no drought)}$$

$$L(\mathbf{X}(\mathbf{s},t)) = \mathrm{E}\left(Y(\mathbf{X}(\mathbf{s},t)) \middle| \mathbf{X}(\mathbf{s},t) \notin D\right) - Y(\mathbf{X}(\mathbf{s},t)) \quad \text{if} \quad \mathbf{X}(\mathbf{s},t) \in D \text{ (there is drought)}$$

$\mathrm{E}\left(Y(\mathbf{X}(\mathbf{s},t)) \middle| \mathbf{X}(\mathbf{s},t) \notin D\right)$: conditional expectation of yield if there is no drought

Then risk can be expressed as,

$$Risk = \mathrm{E}\left(L(\mathbf{X})\right) = \left(\mathrm{E}\left(Y(\mathbf{X}) \middle| \mathbf{X} \notin D\right) - \mathrm{E}\left(Y(\mathbf{X}) \middle| \mathbf{X} \in D\right)\right) \cdot P_D$$

Special case: definition of set D by an SPI

$$D = \{ \mathbf{x} \mid SPI < C_P \} \text{ where } \mathrm{P}\left(SPI < C_P \right) = \Phi(C_P) = P_D$$

and $\Phi(x)$ is the standard normal distribution function.

In this project SPI was used for drought identification.

However other optional drought index or drought definition can be used!

Estimation of loss function based on estimated yield function and SPI

Let $\hat{Y}(\mathbf{X}(\mathbf{s},t))$ $(t=1,..,n)$ be an estimation of the yield function based on a sample.

Estimation of loss function using estimated yield function and SPI:

$$\hat{L}(\mathbf{X}(\mathbf{s},t))=0 \quad \text{if} \quad SPI(\mathbf{s},t)\geq C_P \text{ (there is no drought)}$$

$$\hat{L}(\mathbf{X}(\mathbf{s},t))=\left(\frac{1}{n-n_D}\sum_{SPI(\mathbf{s},t)\geq C_P}\hat{Y}(\mathbf{X}(\mathbf{s},t))\right)-\hat{Y}(\mathbf{X}(\mathbf{s},t)) \quad \text{if} \quad SPI(\mathbf{s},t)<C_P \text{ (there is drought)}$$

$$n_D=\sum_{SPI(\mathbf{s},t)<C_P}1 \ : \ \text{frequency of drought}$$

Estimation of drought risk based on estimated yield function and SPI

$$\hat{R}isk(\mathbf{s}) = \frac{1}{n}\sum_{t=1}^{n}\hat{L}(\mathbf{X}(\mathbf{s},t)) = \left(\left(\frac{1}{n-n_D}\sum_{SPI(\mathbf{s},t)\geq C_P}\hat{Y}(\mathbf{X}(\mathbf{s},t))\right) - \left(\frac{1}{n_D}\sum_{SPI(\mathbf{s},t)<C_P}\hat{Y}(\mathbf{X}(\mathbf{s},t))\right)\right)\cdot\hat{P}_D$$

where $\hat{P}_D = \frac{n_D}{n}$ is the estimated probability of drought.

It is an aesthetic formula, thus it must be good!

Remark about the nature of Risk

The structure of Drought Risk: $Risk_D = \mathrm{E}(L_D) = \mathrm{E}(L_D \mid D) \cdot P_D$

D: drought L_D: loss P_D: probability of drought

Different definitions of drought: D_1, D_2

Maybe: $P_{D_1} < P_{D_2}$ but $\mathrm{E}(L_{D_1} \mid D_1) > \mathrm{E}(L_{D_2} \mid D_2)$ and $Risk_{D_1} = \mathrm{E}(L_{D_1}) \approx \mathrm{E}(L_{D_2}) = Risk_{D_2}$

Similarly to the Roulette

If we have 1 jeton then for all the bets: $Risk = \mathrm{E}(Loss) = \frac{1}{37}$

Single number bet: $Risk = \mathrm{E}(Loss) = 1 \cdot \frac{36}{37} - 35 \cdot \frac{1}{37} = \frac{1}{37}$

Red or black bet: $Risk = \mathrm{E}(Loss) = 1 \cdot \frac{19}{37} - 1 \cdot \frac{18}{37} = \frac{1}{37}$

Estimation of the relative yield function that was used

Relative yield in percent: $\mathrm{Y}_{rel}(\mathbf{s},t)=\dfrac{\mathrm{Y}(\mathbf{s},t)-\mathrm{E}(\mathrm{Y}(\mathbf{s},t))}{\mathrm{E}(\mathrm{Y}(\mathbf{s},t))}\cdot 100\%$

Using the equality $\mathrm{E}(\mathrm{Y}_{rel}(\mathbf{s},t))\equiv 0$ the following regression model was applied,

$$\hat{Y}_{rel}(\mathbf{X}(\mathbf{s},t))=\alpha_1\cdot PI(\mathbf{s},t)+\alpha_2\cdot TI(\mathbf{s},t)$$

using prec. index $PI(\mathbf{s},t)=\ln X_1(\mathbf{s},t)-E_{\ln,1}(\mathbf{s})$ and temp. index $TI(\mathbf{s},t)=X_2(\mathbf{s},t)-E_2(\mathbf{s})$, where $X_1(\mathbf{s},t)$ is precipitation sum and $X_2(\mathbf{s},t)$ is temperature mean for a given 3 or 6 months period, and $E_{\ln,1}(\mathbf{s})$, $E_2(\mathbf{s})$ are the spatial expected values.

The common coefficients α_1,α_2 can be estimated by the method of least squares.

At the risk estimation the $SPI(X_1(\mathbf{s},t))$ can be applied. Then we can evaluate the connection of the different period SPI series with the real drought loss.

The regression coefficients α_1,α_2 were estimated for several 3 or 6 months periods and four plants: maize, wheat, rape and barley.

Algorithm for drought risk assessment applied in the project

1. Selection of the relevant meteorological variables $\mathbf{X}(\mathbf{s},t)$.
   Monthly precipitation and temperature series can be selected.

2. Collecting sample for the crop yield $\mathrm{Yield}(\mathbf{s},t)$ and the meteorological variables $\mathbf{X}(\mathbf{s},t)$.

3. Estimation of the relative crop yield function $\hat{Y}_{rel}\left(\mathbf{X}(\mathbf{s},t)\right)$ that is a regression of the relative crop yield on the meteorological variables.

4. On the basis of the results of item 3 developing software for calculation of the following series and estimations in case of given meteorological data series $\mathbf{X}(\mathbf{s},t)$ $(t=1,..,n)$:

- Estimated relative crop yield functions $\hat{Y}_{rel}\left(\mathbf{X}(\mathbf{s},t)\right)$ $(t=1,..,n)$ according to item 3.
- Several $SPI(\mathbf{s},t)$ $(t=1,..,n)$ series.
- Estimated meteorological drought risk $\hat{R}isk(\mathbf{s})$.

5. Applying software, risk calculations on station and gridded climate data series.
   Mapping of risk for CarpatClim, DanubeClim area based on gridded data series.

6. Mapping of risk outside CarpatClim, DanubeClim area based on station data series and possibly other gridded data series (e.g. E-Obs).

Risk Estimation of Drought
(R E D v1.01)

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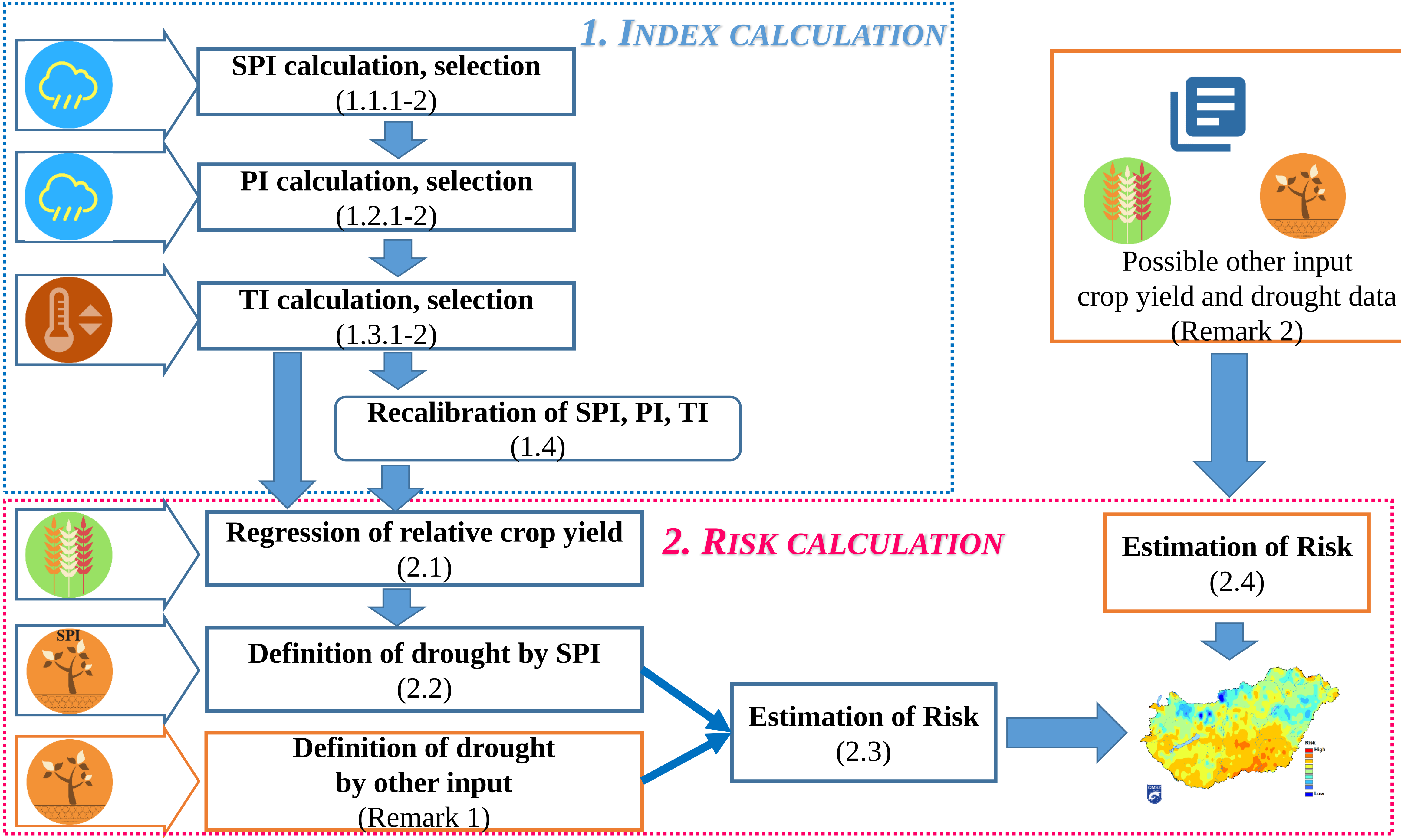
Some Explanation

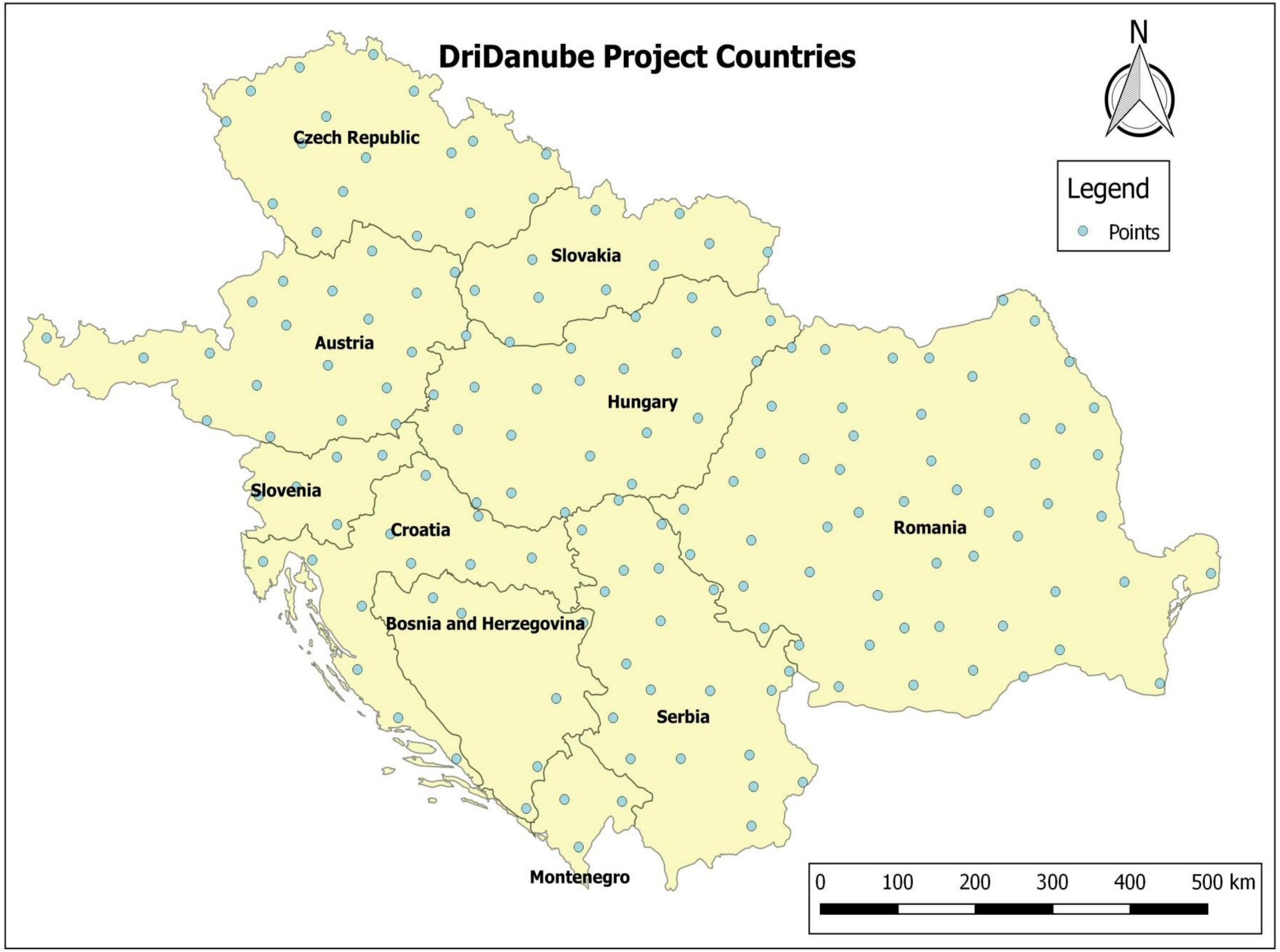
I retired from the Hungarian Meteorological Service.

I continue my activity in VARIMAX Limited Partnership.

Software **RED**: Risk Estimation of Drought

Movie **RED**: Retired, Extremely Dangerous


1. INDEX CALCULATION
SPI calculation, selection (1.1.1-2)
PI calculation, selection (1.2.1-2)
TI calculation, selection (1.3.1-2)
Recalibration of SPI, PI, TI (1.4)
Possible other input crop yield and drought data (Remark 2)
2. RISK CALCULATION
Regression of relative crop yield (2.1)
SPI
Definition of drought by SPI (2.2)
Definition of drought by other input (Remark 1)
Estimation of Risk (2.3)
Estimation of Risk (2.4)
Risk
High
Low


DriDanube Project Countries
N
Legend
Points
Czech Republic
Slovakia
Austria
Hungary
Romania
Slovenia
Croatia
Bosnia and Herzegovina
Serbia
Montenegro
0
100
200
300
400
500 km

CarpatClim and DanubeClim database

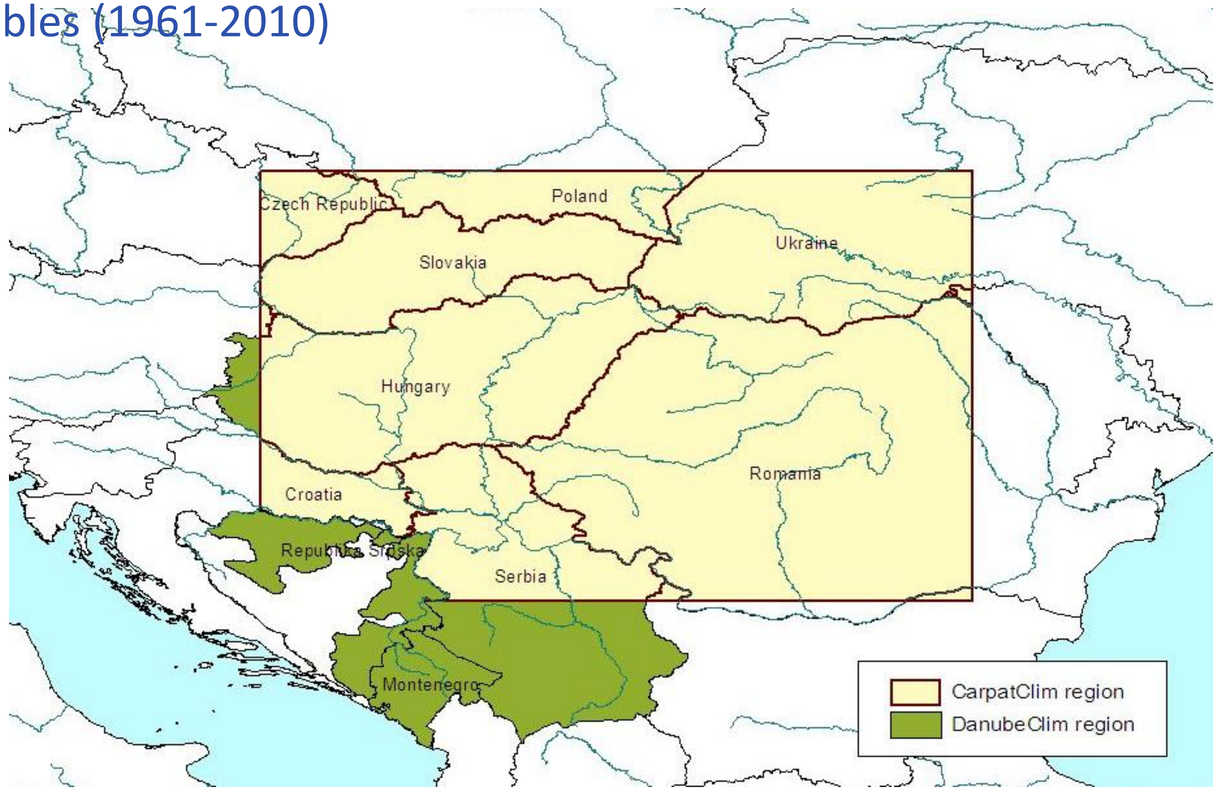
CARPATCLIM
Climate of the Carpathian Region

Daily gridded data series for basic meteorological variables (1961-2010)

Spatial resolution: 0.1°

Methodology: MASH for homogenization,
MISH for gridding

Project of JRC (2011-2013) (10 participants)

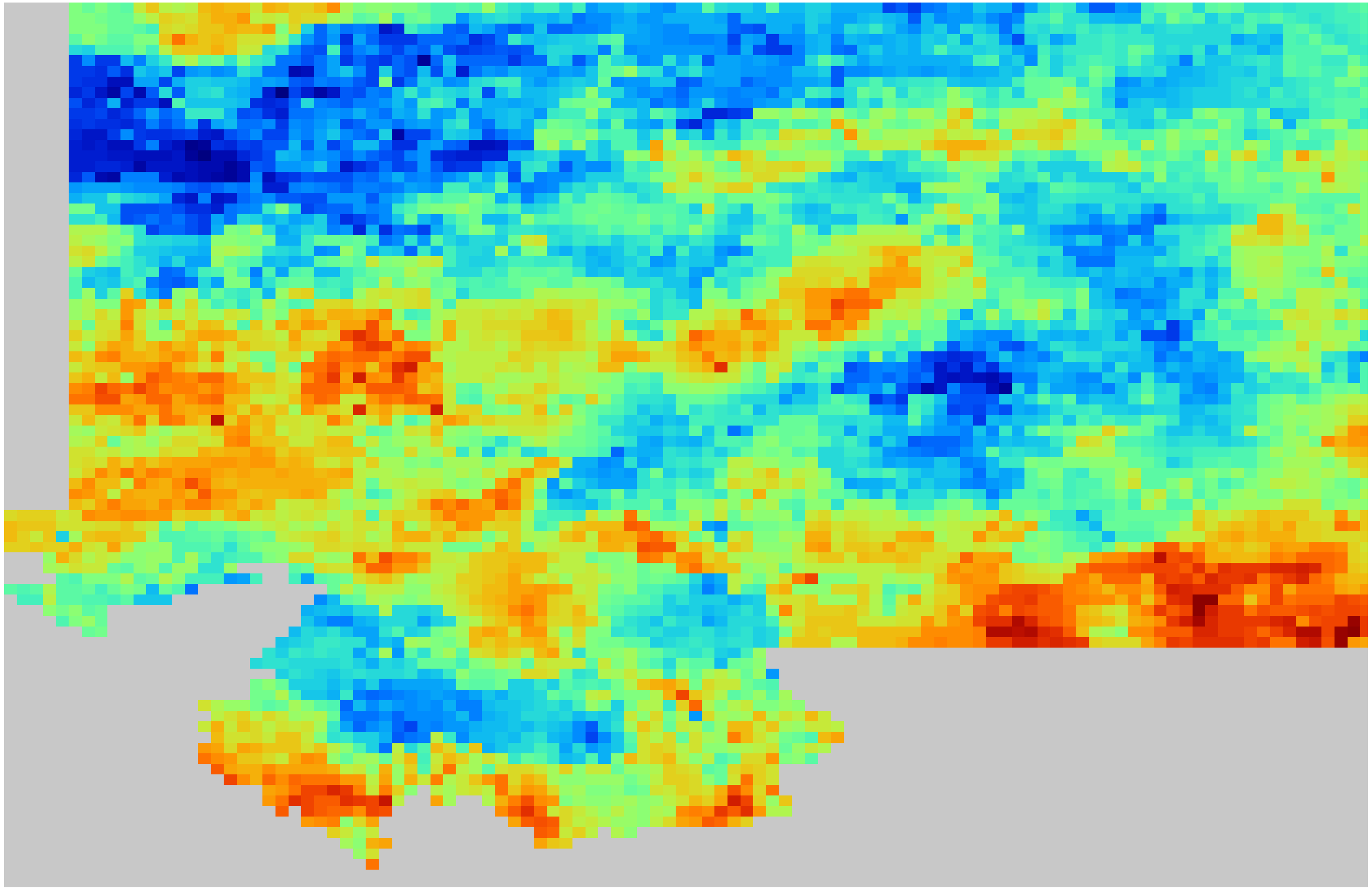


DANUBECLIM
Climate of the Danube Region

Same methods: MISH-MASH

Bilateral contracts with JRC

Risk Map for CarpatClim and DanubeClim area



Risk Map for Maize (period: 5-7, drought probability: 0.2)

Based on gridded data series

Spatial resolution 0.1° (6913 grid)

Output risk.res values presented

There is no royal road! (Archimedes)

Thank you for your attention

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