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Cyclic variation in the precipitation conditions of the Mátra-Bükkalja region and the development of a prognosis method

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Abstract—The cycle properties of the annual average, absolute maximum, and absolute minimum precipitation values have been calculated from precipitation data the Mátra and Bükk regions. The cycle parameters of annual average and annual absolute maximum precipitation values have been determined using the data of a shorter 34-year (1970–2006) and a longer 53-year (1960–2012) period (38 precipitation measurement stations) through the determination of the parameters of frequency, amplitude, and phase with an analytic version of the discrete Fourier transform (DFT), and the values obtained on the basis of the two periods have been compared. Using prognosis parameters, a prognosis until 2025 has been made. Then, the regression function of the variation in time of average and absolute maximum precipitation values has been determined on the basis of actual and prognosticated data for the whole period (1960–2025).

Key-words: Mátra-Bükkalja region, precipitation, cyclic variation, prognosis method

1. Introduction

The analysis of precipitation data in the Mátra-Bükkalja region between the years 1960 and 2012 has given the result that that both the 53-year average values of specific precipitation and the annual absolute maximum values of the measured values for the 38 precipitation measurement stations (settlements) show cylicity for both the 3–5 years and longer periods (*INNOCENTER*, 2013a; *Kovács*, 2014). Minimum and maximum 'local' values recur for both annual average and annual maximum values. With the cyclic variation of annual precipitation values, annual average precipitation displayed constancy around the 600 mm/year value in both the Mátra and Bükkalja regions even on the basis of the combined set of data. With respect to annual absolute maximum and minimum values, regarding these parameters as indicators of extreme weather, plenty of precipitation or years of drought, the data of 53 years showed a decreasing tendency.

In the present paper, the cycle parameters of the average and absolute maximum precipitation values are calculated using the data sets reported in *INNOCENTER* (2013a) and *Kovács* (2014), analysing the precipitation data of the region investigated (Mátra-Mátraalja, Bükk-Bükkalja) and developing a calculation method of cycle parameters as a research task in the Carpathian Basin (*Szűcs*, 2012). Based on this, a prognosis is made for the period until 2025.

2. Theoretical basis of analysis and calculation, the Fourier transform

In the interpretation of frequency, amplitude. and phase, a 2π periodical $\cos(t)$ function has been taken as starting point, where $T = 2\pi$ is the period length of the function. Next, the argument of the function has been transformed (*Meskó*, 1984; *Turai*, 1983):

$$\cos(t) = \cos\left(\frac{2\pi}{2\pi}t\right) = \cos\left(\frac{2\pi}{T}t\right) = \cos\left(2\pi\frac{1}{T}t\right) = \cos(2\pi ft)$$

The rate expressing the density of periods (period density or with the commonly used term, frequency) is

$$f = \left(\frac{1}{T}\right).$$

If t stands for length in space, then frequency gives the number of periods per unit of spatial length for the given direction. Spatial frequency is called wave number.

Multiplying the $\cos(2\pi ft)$ function with factor A and shifting its maximum by Δt, after writing up

$$\cos(2\pi f[t + \Delta]),\ \text{factor } A$$

is called amplitude. In the case of a monofrequency periodical signal, the amplitude equals half of the difference between the maximum (F_{max}) and minimum (F_{min}) of signal value:

$$A = \frac{F_{\max} - F_{\min}}{2}.$$

After a further transformation of the argument of the cosine function, the following formula can be written:

$$A\cos(2\pi f[t + \Delta t]) = A\cos(2\pi ft + 2\pi f\Delta t) = A\cos(2\pi ft + 2\pi \frac{\Delta t}{T}) = A\cos(2\pi ft + \varphi).$$

The quantity φ, thus introduced, is called phase (phase angle). the absolute phase shows the part of the phase length (phase time or wavelength) the maximum of the signal has shifted with in relation to the origin ($t = 0$). As it can be seen in *Fig. 1*, in the case of $\Delta t = 0$, the maximum shifts to the, left while in the case of $\Delta t < 0$ to the right of the origin. Absolute phase can be given in both radians and degrees:

$$\varphi = 2\pi \frac{\Delta t}{T} \quad \text{[rad]} \qquad \varphi = 360 \frac{\Delta t}{T} \quad \text{[degrees]}.$$

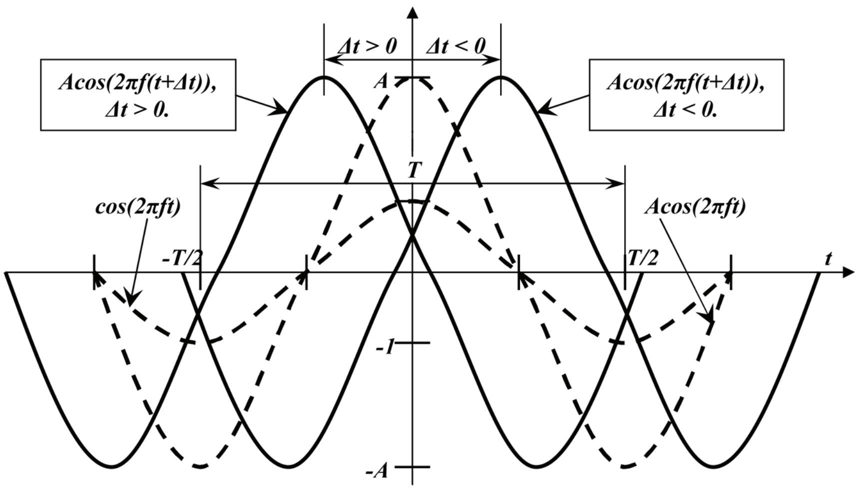

Fig. 1. The interpretation of the absolute phase.

Relative phase ($\Delta\varphi$) is interpreted between two signals and shows that in relation to the maximum of one of two signals of identical frequency, what part of the period length the maximum of the other signal has shifted with. As it can be seen in *Fig. 2*, the two signals are *x(t)* and *y(t)* while the difference of the maximums of the two signals Δt_{xy}. The relative phase between the two signals can also be calculated:

$$\Delta\varphi_{xy} = 2\pi \frac{\Delta t_{xy}}{T} \text{ [rad]} \qquad \Delta\varphi_{xy} = 360 \frac{\Delta t_{xy}}{T} \text{ [degrees].}$$

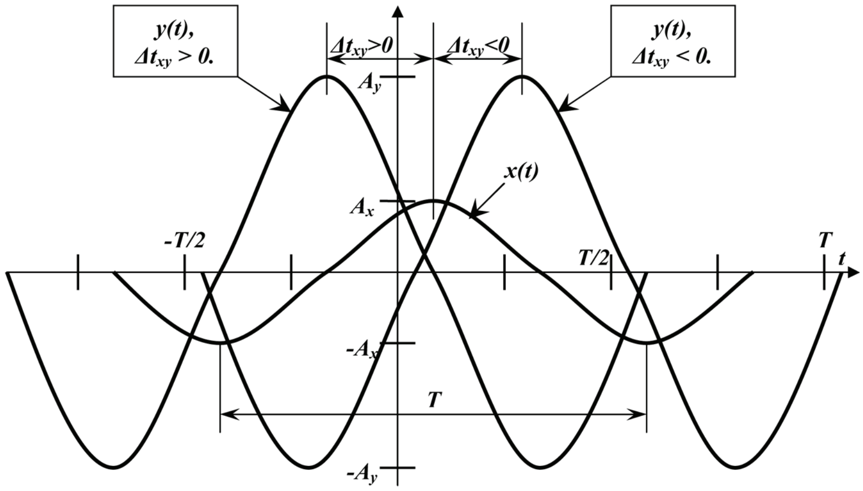

Fig. 2. The interpretation of the relative phase.

The relative phase can also be calculated as the difference of the absolute phases of the two signals:

$$\Delta\varphi_{xy} = \varphi_y - \varphi_x .$$

With the help of the Fourier transform, signals can be transferred from the space-time domain into the frequency domain. During the process, the mappings of signals in the frequency domain are called Fourier spectra.

Working with harmonic functions (cos$(2\pi ft)$, sin$(2\pi ft)$) in the analytic Fourier transform, a complex Fourier spectrum is obtained, which can be divided into a real and an imaginary part. The Re[$F(f)$] real part of the spectrum can be written up with a real cosine transformation

$$\mathrm{Re}[F(f)] = \int_{-\infty}^{+\infty} f(t)\cos(2\pi ft)dt , \tag{1}$$

while its imaginary part with a real sine transformation is

$$\mathrm{Im}[F(f)] = \int_{-\infty}^{+\infty} f(t)\sin(2\pi ft)dt . \tag{2}$$

The complex Fourier spectrum can be written up with two real spectra:

$$F(f) = \mathrm{Re}[F(f)] + j\,\mathrm{Im}[F(f)].$$

The real spectrum gives the weights of the cosine components falling into a frequency band unit around any f frequency, while the imaginary spectrum gives the weights of the sine components for the formation of the signal.

The *F(f)* complex spectrum can also be defined in an exponential form by the introduction of two other real spectra:

$$F(f) = A(f) e^{j\Phi(f)}.$$

The *A(f)* spectrum, thus introduced, is called amplitude spectrum, while the $\phi(f)$ spectrum is called phase spectrum. The amplitude spectrum gives the weight in the formation of the signal of the harmonic component falling into a frequency band unit around any f frequency, while the phase spectrum shows the part of the period length the maximum of this harmonic component shifts with in relation to the maximum of base function $\cos(2\pi ft)$, taken at point $t = 0$.

The amplitude and phase spectra are the following in the knowledge of real and imaginary spectra with the help of the correlations yielded by *Fig. 3*:

$$A(f) = \sqrt{(\mathrm{Re})^2[F(f)] + (\mathrm{Im})^2[F(f)]}$$

$$\phi(f) = arctg\frac{\mathrm{Im}[F(f)]}{\mathrm{Re}[F(f)]}$$

Real and imaginary spectrum values can also be calculated from amplitude and phase spectra:

$$\mathrm{Re}[F(f)] = A(f)\cos[\phi(f)]$$
$$\mathrm{Im}[F(f)] = A(f)\sin[\phi(f)]$$

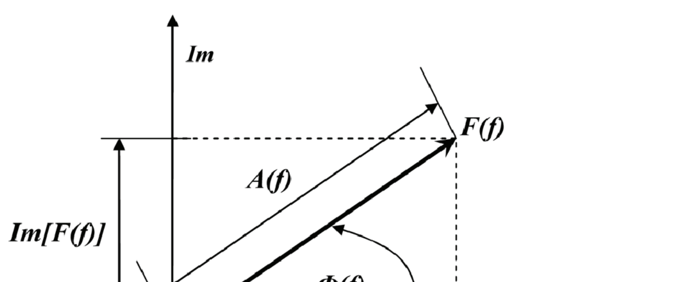

Fig. 3. Plotting of Fourier spectra in a complex plane.

3. Spectral analysis

In the search for the deterministic periodic components, the spectrum of the $\Delta y(t)$ deviations from the ($\overline{Y}$) expected values has been investigated with the following correlations:

$$\Delta y(t) = y(t) - \hat{Y},$$

$$Y(f) = \int_{-\infty}^{+\infty} \Delta y(t) e^{-j2\pi ft} dt.$$

The period lengths of the deterministic periodic components to be found in the stochastic signal are given by the reciprocal values of the ($f_{1,max}$, $f_{2,max}$, …, $f_{N,max}$) frequencies belonging to the maximums of the *A(f)* amplitude density spectrum of the *Y(f)* spectrum:

$$T_1 = \frac{1}{f_{1,\max}},$$

$$T_2 = \frac{1}{f_{2,\max}},$$

$$\dots$$

$$T_N = \frac{1}{f_{N,\max}},$$

where N is the number of deterministic periodic components (the number of the maximums of the *A(f)* spectrum).

It can be calculated from the $\phi(T_i)$ values of the phase-density spectrum belonging to the given period time, what $\Delta t(T_i)$ time the maximum of the given component of any T_i (i = 1, 2, …, N) period time has shifted in relation to the starting year (1973) of data registration:

$$\Delta t(T_i) = T_i \frac{\Phi(T)_i}{2\pi} \quad [\text{radian}],$$

or

$$\Delta t(T_i) = T_i \frac{\Phi(T)_i}{360} \quad [\text{degree}].$$

The A_i amplitudes of a component of any T_i period time are given by the values of *A(f)* amplitude density:

$$A_i = A(T_i).$$

Figure A_i gives the amplitude of the deterministic component with T_i period time.

Let $A(f)_{\max}$ denote the maximum of the $A(f)$ amplitude density spectrum. The relative amplitude density spectrum normed to maximum value ($A(f)_{\mathrm{rel}}$) as the percentage of maximum value can be calculated as follows:

$$A(f)_{rel} = \frac{A(f)}{A(f)_{\max}}.100[\%].$$

Relative amplitude density spectrum values show percentage of the amplitude density of any given component of T = 1/f period time in the maximum amplitude density.

4. Spectral analysis of the variation of annual precipitation amount on the basis of Mátra Bükkalja precipitation data

In INNOCENTER (2013b) the cycle properties of the variation in time of precipitation have been investigated on the basis of the territorial average values of precipitation data in the years 1960–2012 in 23 settlements/precipitation measurement stations in the Mátra-Mátraalja region and 15 settlements/ precipitation measurement stations in the Bükk-Bükkalja region. *Table 1* shows the average annual precipitation values and the annual absolute maximum precipitation values on the basis of the data of the two regions and combined data. In order to assess the effect of the registration period on results, cycle properties have been calculated for a shorter (1973–2006, 34 years) and a longer (1960–2012, 53 years) period. (Yearbook of the Hydrographical Service of Hungary 1960–2005.)

4.1. The results of spectral analysis on the basis of precipitation data for the years 1973–2006

In the spectral analysis of the precipitation data, the registration time (T_{reg} was 33 years for end-sampling periods and the 34 years for middle-sampling periods. The sampling rate (Δt) was 1 year, while the number of samples was 34.

Analyses have been performed with an analytic version of the discrete Fourier transform (DFT) (*Turai*, 1983). The complex amplitude density spectra of the function of annual precipitation values have been determined as the function of discrete period time values. Of the four real spectra describing the complex spectrum (real spectrum, imaginary spectrum, amplitude spectrum, and phase spectrum), amplitude spectra are presented. In the plotting, logarithmic linear scale has been chosen to illustrate spectrum maximums more clearly.

Table 1. Precipitation data in the Mátra-Bükkalja region as the function of time

Number of settlements / measurement points		**Years**												
		1960	**1961**	**1962**	**1963**	**1964**	**1965**	**1966**	**1967**	**1968**	**1969**	**1970**	**1971**	**1972**
Mátra	**Average**	727	506	568	681	747	868	814	538	478	737	861	455	636
	Absolute maximum	1016	584	724	923	1012	1098	1029	610	599	899	1080	559	789
Bükk	**Average**	712	436	499	548	693	825	769	569	517	623	891	378	557
	Absolute maximum	811	487	535	599	769	947	956	733	563	764	971	410	674
Mátra Bükk	**Average**	721	476	538	624	724	849	795	551	495	688	874	422	602
	Absolute maximum	1016	584	724	923	1012	1098	1029	733	599	899	1080	559	789

Number of settlements / measurement points		**Years**													
		1973	**1974**	**1975**	**1976**	**1977**	**1978**	**1979**	**1980**	**1981**	**1982**	**1983**	**1984**	**1985**	**1986**
Mátra	**Average**	509	804	607	709	620	576	691	659	504	444	425	655	674	453
	Absolute maximum	712	1061	748	935	748	704	844	821	595	555	504	791	814	566
Bükk	**Average**	453	698	675	659	691	585	652	697	524	423	504	535	594	399
	Absolute maximum	535	735	843	747	781	708	763	740	574	506	581	648	680	438
Mátra Bükk	**Average**	485	759	636	688	650	580	674	676	512	435	459	604	640	430
	Absolute maximum	712	1061	843	935	781	708	844	821	595	555	581	791	814	566

Number of settlements / measurement points		**Years**													
		1987	**1988**	**1989**	**1990**	**1991**	**1992**	**1993**	**1994**	**1995**	**1996**	**1997**	**1998**	**1999**	**2000**
Mátra	**Average**	534	638	611	560	692	393	514	497	718	637	433	654	949	474
	Absolute maximum	654	851	729	613	774	486	603	586	874	751	519	754	1092	563
Bükk	**Average**	499	596	586	492	652	346	457	454	679	576	482	722	874	426
	Absolute maximum	559	613	655	523	694	418	516	509	727	609	535	774	988	474
Mátra Bükk	**Average**	519	620	600	531	675	373	490	479	701	611	454	683	917	453
	Absolute maximum	654	851	729	613	774	486	603	586	874	751	535	774	1092	563

Number of settlements / measurement points		**Years**											
		2001	**2002**	**2003**	**2004**	**2005**	**2006**	**2007**	**2008**	**2009**	**2010**	**2011**	**2012**
Mátra	**Average**	585	639	551	647	751	651	632	657	666	1054	404	434
	Absolute maximum	723	837	674	747	929	733	791	777	736	1195	462	486
Bükk	**Average**	653	591	517	749	741	631	599	625	624	1118	447	466
	Absolute maximum	736	669	564	828	791	711	649	701	709	1153	488	557
Mátra Bükk	**Average**	614	618	536	690	746	642	619	645	658	1079	420	446
	Absolute maximum	736	837	674	828	929	733	791	777	736	1195	488	557

In the calculation of spectra, the spectrum of $\Delta y(t)$ deviations from ($\overline{Y}\,\hat{Y}$) expected values has been determined:

$$\Delta y(t) = y(t) - \overline{Y}\,.$$

The (T_{min}) minimum period time that can theoretically be found in the signal is defined by the Nyquist frequency (f_N).

$$\Delta t = 1 \text{ year}, \qquad f_N = 0{,}5\,\frac{1}{year}, \qquad T_{min} = 2 \text{ years}.$$

As in the case of all the six time series, the 'sampling' time was 1 year, the analysis can only reveal cycles of longer period time than 2 years in the changes everywhere.

In theory, maximum period time (T_{max}) is determined by the registration time (T_{reg}):

$T_{max} = T_{reg}$ – in case of end sampling,

$T_{max} = T_{reg} + \Delta t$ – in case of middle sampling.

Therefore, the maximum period time that can be revealed by analysis is 33 years in case of end-sampling and 34 years in case of middle-sampling

With the data in *Table 1*, both the amplitude spectra of the amplitude density and the relative spectra have been determined. In the latter case, spectra have been normed to maximum spectrum value. In all the six cases – annual average and annual absolute maximum precipitation, – for Mátra, Bükk, and Mátra+Bükk regions, similar amplitude and relative amplitude spectrum functions have been obtained.

The cycle properties of annual average precipitation in the Mátra region are the following on the basis of amplitude peaks, cycle time, and amplitude density:

Major cycles: 1. T_1 = 4.9 years, A_1 = 1243 mm; 2. T_2 = 3.5 years, A_2 = 1195 mm; 3. T_3 = 29.8 years, A_3 = 946 mm; 4. T_4 = 9.9 years, A_4 = 806 mm; minor cycles: 1. T_1 =7.3 years, A_1 = 476 mm; 2. T_2 = 6.3 years, A_2 = 440 mm.

Cycle properties revealed on the basis of Bükk data are, cycle time and amplitude density: major cycles: 1. T_1 = 28.7 years, A_1 = 1216 mm; 2. T_2 = 3.5 years, A_2 = 1064 mm; 3. T_3 = 4.9 years, A_3 = 1035 mm; 4. T_4 = 9.5 years, A_4= 929 mm; minor cycles: 1. T_1 = 7.3 years, A_1 = 541 mm; 2. T_2 = 6.1 years, A_2 = 308 mm.

The combined treatment of Mátra+Bükk data has also revealed 4 major and 2 minor cycles in the variation of annual precipitation values (*Figs. 4* and *5*), cycle time and amplitude density: major cycles 1. T_1 = 5.0 years, A_1 = 1,139 mm; 2. T_2 = 3.5 years, A_2 = =1,119 mm; 3. T_3 = 29.2 years, A_3 = 1,080 mm; 4. T_4 = 9.7 years, A_4 = 860 mm; minor cycles 1. T_1 = 7.4 years, A_1 = 508 mm; 2. T_2 = 6.2 years, A_2 = 310 mm.

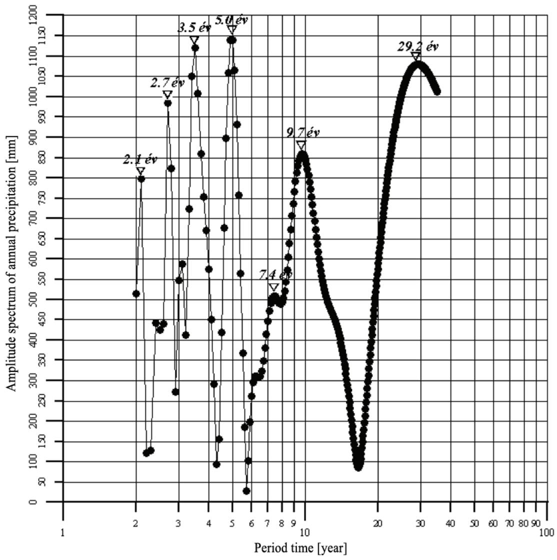

Fig. 4. Amplitude spectrum of annual precipitation int he Mátraalja and Bükkalja regions.(sampling rate = 1 year)

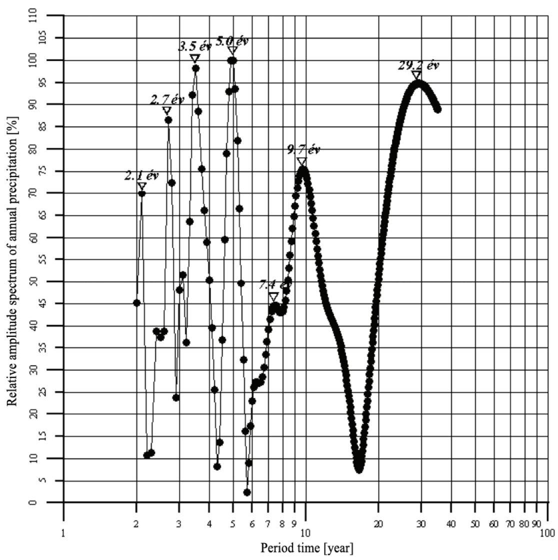

Fig. 5. Relative amplitude spectrum of annual precipitation in the Mátraalja and Bükkalja regions. (sampling rate = 1 year)

Cycle properties that can be revealed on the basis of the amplitude spectrum and relative amplitude spectrum detected in the variation of annual absolute maximum precipitation values, cycle time, and amplitude density for the Mátra region are the following: 1. T_1 = 3.5 years, A_1 = 1561 mm; 2. T_2 = 5.0 years, A_2 = 1434 mm; 3. T_3 = 10.9 years, A_3 = 1352 mm; 4. T_4 = 31.4 years, A_4 = 1262 mm; minor cycles 1. T_1 = 7.5 years, A_1 = 741 mm; 2. T_2 = 6.2 years, A_2 = 474 mm.

Cycle properties of the Bükk region are: major cycles 1. T_1 = 27.0 years, A_1 = 1408 mm; 2. T_2 = 3.4 years, A_2 = 1297 mm; 3. T_3 = 5.0 years, A_3 = 1168 mm; 4. T_4 = 9.7 years, A_4 = 973 mm; minor cycles 1. T_1 = 7.4 years, A_1 = 796 mm; 2. T_2 = 6.2 years, A_2 = 362 mm.

Cycle properties of Mátra and Bükk combined data on the basis of amplitude spectra (*Figs. 6* and *7*), cycle time, and amplitude density are the following: major cycles 1. T_1 = 3.5 years, A_1 = 1482 mm; 2. T_2 = 5.0 years, A_2 = 1413 mm; 3. T_3 = 30.3 years, A_3 = 1256 mm; 4. T_4 =11.1 years, A_4 = 1225 mm; minor cycles 1. T_1 = 7.5 years, A_1 = 734 mm; 2. T_2 = 6.2 years, A_2 = 298 mm.

On the basis of the above results, the following generalizations can be made:

— In the case of the six time series examined with respect to annual precipitation variation, cycles of approximately identical period times can be revealed.

— In the case of all the six time series, there have been found periods of 3.5 years, 5 years, 10–11 years, and 27–31 years as major cycles.
— In all the cases, 6.2-year and 7.3–7.5-year periods appear as minor cycles. (To prove the existence of 27–31-year cycles in a more reliable way, longer data series would be needed.)

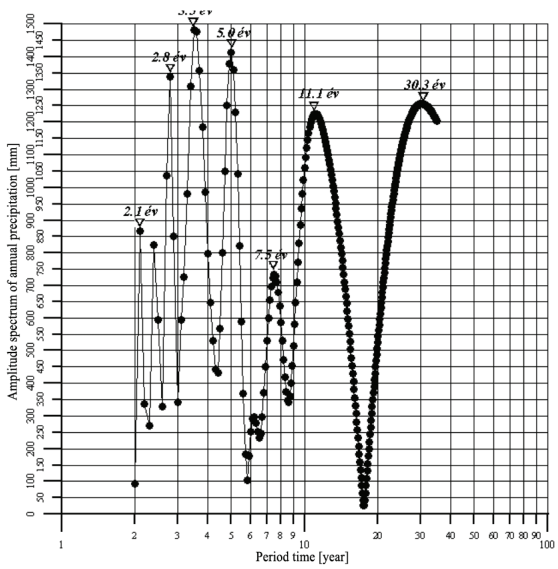

Fig. 6. Amplitude spectrum of annual precipitation in the Mátraalja and Bükkalja regions (sampling rate = 1 year).

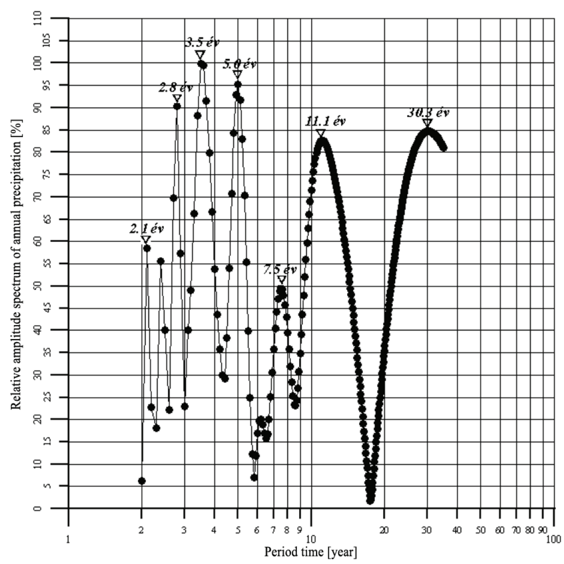

Fig. 7. Relative amplitude spectrum of the absolute maximum of annual precipitation in the Mátraalja and Bükkalja regions (sampling rate = 1 year.)

4.2. The results of spectral analysis on the basis of precipitation data in the years 1960-2012

The registration period is 1960–2012, the length of the registration period (T_{reg}) is 52 years with end sampling and 53 years with middle sampling, sampling rate (Δt) is 1 year, the number of samples is 53. The calculation process has been according to Section 4.1, the maximum period time that the analysis can reveal is T_{max} = 52 years – 53 years.

On the basis of amplitude peaks, the following precipitation cycles can be revealed for the Mátra annual precipitation values, cycle time, and amplitude density: major cycles T_1 = 5.0 years, A_1=2765 mm; 2. T_2 = 3.6 years, A_2 = 2074 mm; 3. T_3 = 41.1 years, A_3 = 1555 mm; 4. T_4 = 10.7 years, A_4 = 1494 mm; minor cycles 1. T_1 = 6.4 years, A_1 = 1101 mm; 2. T_2 = 5.7 years, A_2 = 1027 mm; 3. T_3 = 8.6 years, A_3 = 675 mm; 4. T_4 = 14.3 years, A_4 = =642 mm; 5. T_5 = 7.4 years, A_5 = 577 mm; 6. T_6 = 19.8 years, A_6 = 456 mm.

In the Bükkalja region, the following cycles can be revealed in the variation of annual precipitation values on the basis of amplitude spectrum and relative amplitude spectrum, cycle time, and amplitude density: major cycles 1. T_1 =

5.0 years, A_1 = 2567 mm; 2. T_2 = 38.6 years, A_2 = 1759 mm; 3. T_3 = 10.5 years, A_3 = 1747 mm; 4. T_4 = 3.6 years, A_4 = 1719 mm; minor cycles 1. T_1 = 5.7 years, A_1 = 1413 mm; 2. T_2 = 6.5 years, A_2 = 1220 mm; 3. T_3 = =14.2 years, A_3 = 753 mm; 4. T_4 = 7.5 years, A_4 = 552 mm; 5. T_5 = 8.4 years, A_5 = 504 mm; 6. T_6 = 19.8 years, A_6 = 323 mm.

The combined treatment of Mátra and Bükk data also reveals 4 major and 6 minor cycles on the basis of annual precipitation values (*Figs. 8* and *9*), cycle time and amplitude density: major cycles: 1. T_1 = 5.0 years, A_1 = 2685 mm; 2. T_2 = 3.6 years, A_2 = 1928 mm; 3. T_3 = 40.4 years, A_3 = 1635 mm; 4. T_4 = 10.6 years, A_4 = 1587 mm; minor cycles 1. T_1 =5.7 years, A_1 = 1188 mm; 2. T_2 = 6.4 years, A_2 = 1151 mm; 3. T_3 = 14.2 years, A_3 = 669 mm,;4. T_4 = 8.5 years, A_4 = 592 mm; 5. T_5 = 7.4 years, A_5 = 577 mm; 6. T_6 = 20.0 years, A_6 = 383 mm.

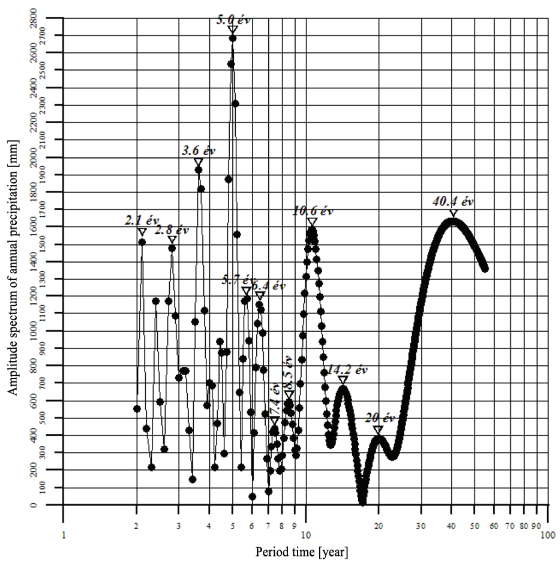

Fig. 8. Amplitude spectrum of annual precipitation in the Mátraalja and Bükkalja regions (sampling rate = 1 year).

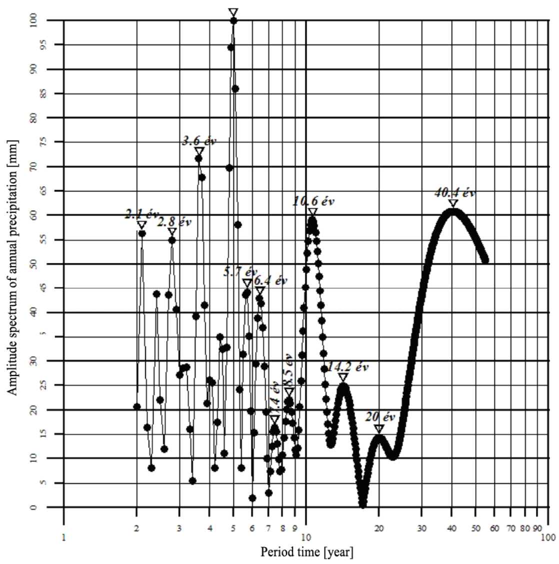

Fig. 9. Relative amplitude spectrum of annual precipitation in the Mátraalja and Bükkalja regions (sampling rate = 1 year).

The analysis of the absolute maximum values of annual precipitation reveals the following cycle properties on the basis of Mátra data, cycle time, and amplitude density: major cycles 1. T_1 = 5.0 years, A_1 = 3306 mm; 2. T_2 = 3.6 years, A_2 = 2656 mm; 3. T_3 = 45.6 years, A_3 = 2119 mm; 4. T_4 = 10.8 years, A_4 = 1806 mm; minor cycles 1. T_1 = 5.6 years, A_1 = 1319 mm; 2. T_2 = 6.4 years, A_2 = 1191 mm; 3. T_3 = 13.9 years, A_3 = 1044 mm; 4. T_4 = 7.3 years, A_4 = 1046 mm; 5. T_5 = 8.6 years, A_5 = 814 mm; 6. T_6 = 19.8 years, A_6 = 722 mm.

Similarly, 4 major and 6 minor cycles can be revealed on the basis of the Bükk-Bükkalja absolute maximum precipitation data, cycle time and amplitude density: major cycles 1. T_1 = 5.0 years, A_1 = 2646 mm; 2. T_2 = 38.6 years, A_2 = 2138 mm; 3. T_3 = 10.5 years, A_3 = 2024 mm; 4. T_4 = 3.6 years, A_4 = 1758 mm; minor cycles 1. T_1 = 5.6 years, A_1 = 1434 mm; 2. T_2 = 6.4 years, A_2 = 1351 mm;

3. T_3 = 14.0 years, A_3 = 885 mm; 4. T_4 = 8.4 years, A_4 = 883 mm; 5. T_5 = 7.3 years, A_5 = 445 mm; 6. T_6 = 19.4 years, A_6 = 454 mm.

Cycle properties of absolute maximum precipitation values cycle time, and amplitude density in the combined assessment of the Mátra+Bükk region (*Figs. 10* and *11*) are the following: major cycles 1. T_1 = 5.0 years, A_1 = 3,168 mm; 2. T_2 = 3.6 years, A_2 = 2468 mm; 3. T_3 = 46 years, A_3 = 2271 mm; 4. T_4 = 10.7 years, A_4 = 1842 mm; minor cycles 1. T_1 = 5.7 years, A_1 = 1273 mm; 2. T_2 = 6.4 years, A_2 = 1127 mm; 3. T_3 = 13.7 years, A_3 = 982 mm; 4. T_4 = 8.5 years, A_4 = 721 mm; 5. T_5 = 7.3 years, A_5 = 726 mm; 6. T_6 = 19.6 years, A_6 = 714 mm.

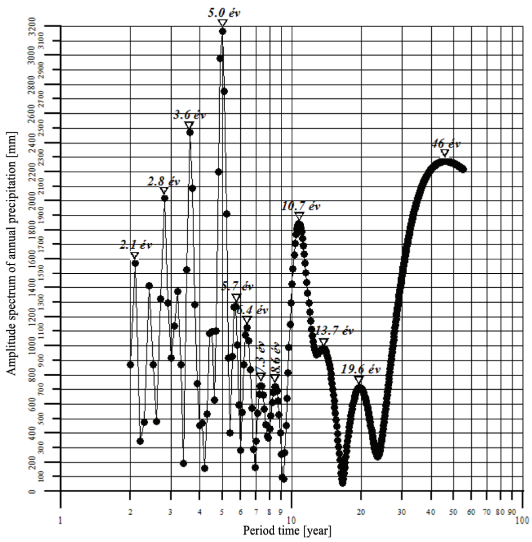

Fig. 10. Amplitude spectrum of annual precipitation in the Mátraalja and Bükkalja regions (sampling rate = 1 year) .

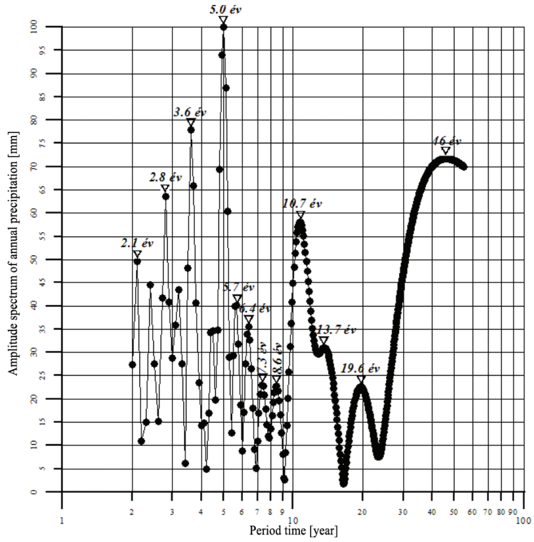

Fig. 11. Relative amplitude spectrum of the absolute maximum of annual precipitation in the Mátraalja and Bükkalja regions (sampling rate = 1 year) .

From the cycle properties determined on the basis of the data of precipitation time series of 53 years, the following generalizations can be made:

— Cycles of nearly identical period time can be revealed on the basis of the six time series investigated with respect to annual precipitation variation.

— In the case of all the six time series, the 3.6-year, the 5-year, the 10.5–10.8-year and 38.6–46-year periods appear as major cycles.

— In all the cases, the 5.6–6.7-year, 6.4-year, 7.3–7.5-year, 8.4–8.6-year, 13.7–14.3-year and 19.4–20-year periods appear as minor cycles.

The comparison of the cycle time data of the major and minor cycles revealed on the basis of the two time series of different lengths (34 years and 53 years) has yielded the following results:

— With all the three data groups, the number of major cycles that can be revealed on the basis of both time series is the same: four.

— In the case of the shorter time series, 2 minor cycles have been found for all the three data groups, while for the longer time series (53 years), 6 minor cycles have been revealed.

— With the shorter, generally maximum 10-year cycle times, practically identical/equivalent cycle time has been revealed for both the major and minor cycles, namely, in Mátra: 3.5–3.6 years, 4.9–5.0 years, 9.9–10.7 years, 6.3–6.4 years, 7.3–7.4 years,in Bükk: 3.5–3.6 years, 4.9–5.0 years, 9.5–10.5 years, in Mátra+Bükk: 3.5–3.6 years, 5.0–5.0 years, 9.7–10.6 years, 6.2–6.4 years, 7.4–7.4 years.

— In all the three areas, it has been identically found for longer cycle times (above 30 years) that on the basis of the 34-year time series, a shorter major cycle time, while on the basis of longer time series, a longer major cycle time has been revealed, namely, in Mátra: 29.8 years, 41.1 years, in Bükk: 28.7 years, 38.6 years, in Mátra+Bükk: 29.2 years, 40.4 years.

The differences found in the latter case confirm the former observation that for a long-time prognosis, a time (data) series longer than 50 years is required.

5. Determination of prognosis values

On the basis of Sections 2 and 3, including the summary of the basics of spectral data processing, the *y(t)* time series of precipitation values can be restored through the ’use’ of the *A(f)* amplitude density and $\phi(f)$ phase density spectra, defined in the previous analyses:

$$y(t) = \overline{Y} + \int_{-f_N}^{+f_N} A(f) e^{j[2\pi ft + \phi(f)]} df \,,$$

where f_N is the Nyquist frequency and it equals to 0.5 year^{-1}.

As the Fourier spectrum is even, the former equation can also be written up in the following form:

$$y(t) = \overline{Y} + \int_{o}^{+f_N} A(f) e^{j[2\pi ft + \phi(f)]} df \,.$$

With the use of the T_i (i = 1,2,…, N=10) period times of major and minor cycles, the A_i (i = 1,2,…, N = 10) amplitude, and the $\phi(T_i)$ (i = 1,2,…, N = 10) phase values, it is possible to define the [$y(t)^{det}$] time series of the amount of precipitation attributable to deterministic causes:

$$y(t)^{\det} = \overline{Y} + \frac{2}{T_{reg}} \sum_{i=1}^{10} A_i \cos\left[\frac{2\pi}{T_i}(t-1960) + \phi(T_i)\right].$$

Using the $\{R_e[F(T_i)]\}$ and $\{I_m[F(T)_i]\}$ values calculated for given T_i period times of real and imaginary spectra, the $\phi(T_i)$ phases of the specific components can be defined with the following correlation:

$$\phi(T_i) = arctg \frac{I_m[F(T_i)]}{R_e[F(T_i)]}.$$

The difference between the *y(t)* actual time series and $y(t)^{det}$ represents the accidental (stochastic) impact.

If $t > 2012$ values are put in the former equation, the amount of precipitation that can be expected in the given years can be estimated (forecast) with extrapolation. It must be added, however, that this estimation would only yield a prognosis of 100% reliability by using spectra calculated from an infinitely large *y(t)* registratum (annual data), which, of course, cannot be expected in the case of the 53 years long time series investigated.

Furthermore, there is a possibility of estimating periodicity with modern statistical methods (analysis with autocorrelation functions, factor and cluster analysis), although these tools would only give similarly precise results as the spectral analysis applied on the basis of data series of several hundred years.

Using the spectrum data in *Fig. 8*, taking into account the impact of the four deterministic major cycles (5, 3.6, 40.4, and 10.6 years) and taking into consideration the impact of the further 6 minor cycles in *Fig. 12* as well as that of the two cycles (2.1 years and 2.8 years) earlier omitted due to aliasing distortion, the prognosis values in *Figs. 13, 14* are obtained. According to *Fig. 4*, the two short cycles are present in the prognosis of annual precipitation values with a relatively high amplitude, above 55%, there has been a spectacular improvement in classic statistical indicators. Deviation (RMS) has decreased from 16.1% and 15.7% to 12.6%, while the correlation coefficient (r) has increased from 0.78 and 0.79 to 0.89.

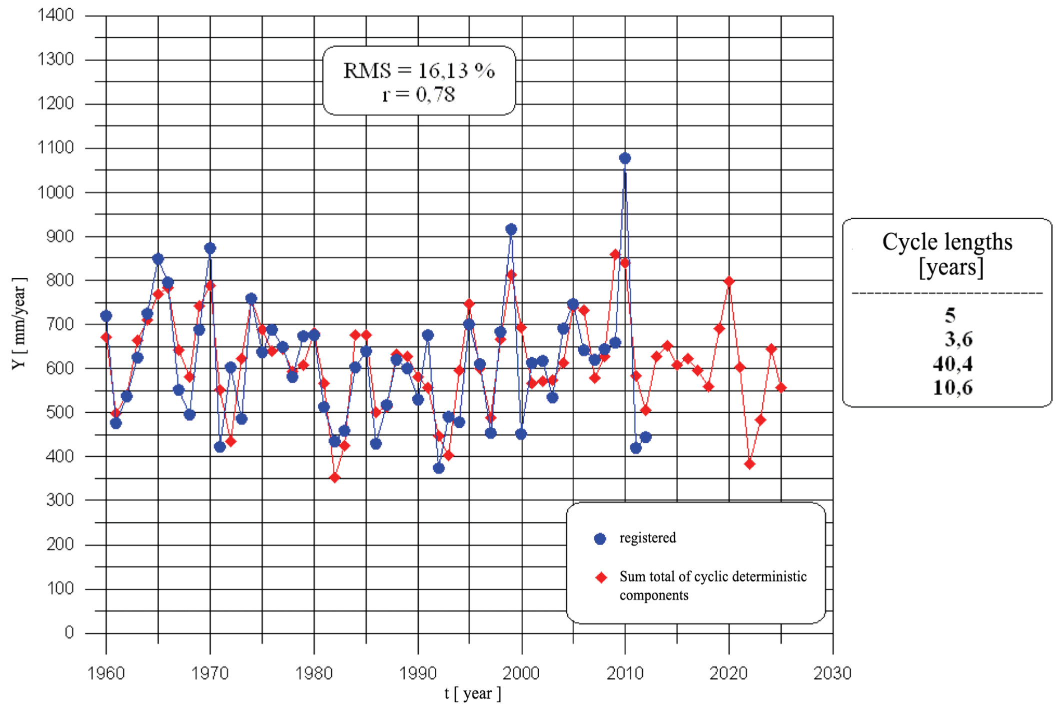

Fig. 12. Annual precipitation value in the Mátraalja and Bükkalja regions. (Prognosticated on the basis of four cyclic components.)

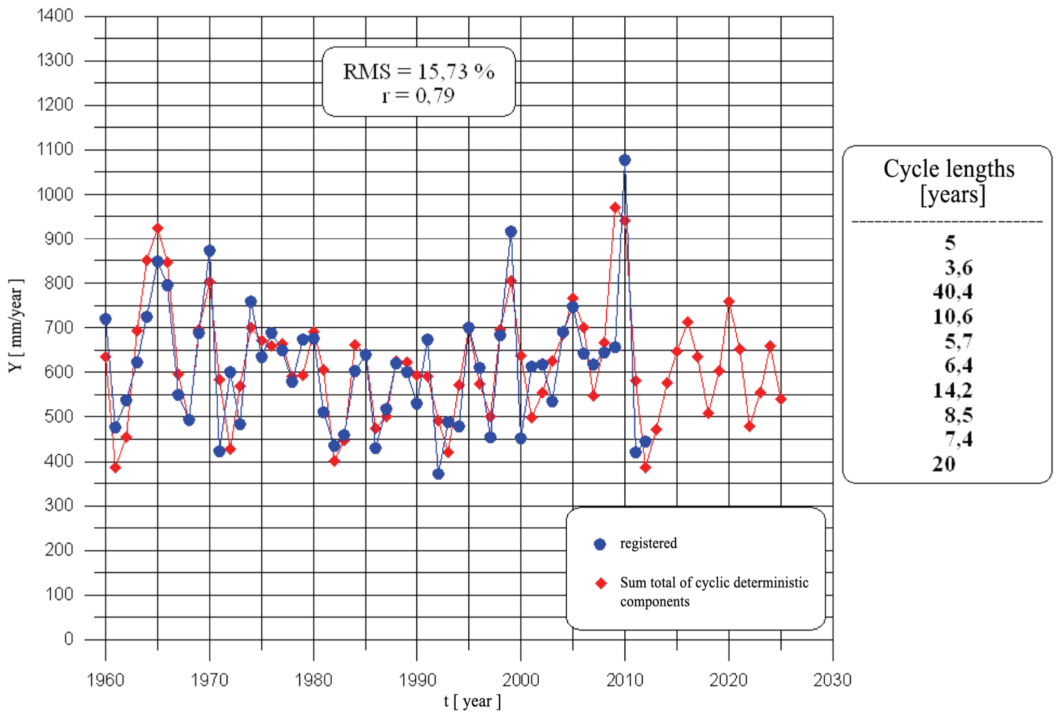

Fig. 13. Annual precipitation value in the Mátraalja and Bükkalja regions. (Prognosticated on the basis of ten cyclic components.)

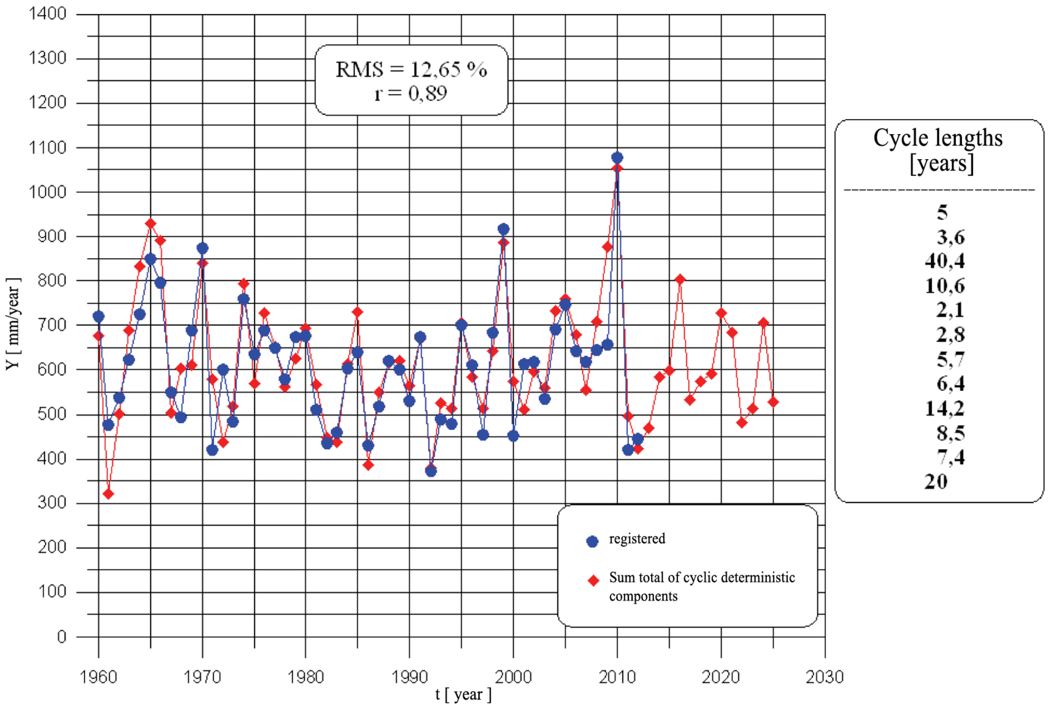

Fig. 14. Annual precipitation value in the Mátraalja and Bükkalja regions. (Prognosticated on the basis of twelve cyclic components.)

The amplitude data in *Fig. 10* and relative amplitude data in *Fig. 11* have been used in the calculation of annual absolute maximum precipitation prognosis. Taking the four deterministic and the further 6+2 cycle properties into account, the absolute maximum precipitation prognosis in *Figs. 15, 16,* and *17* has been obtained.

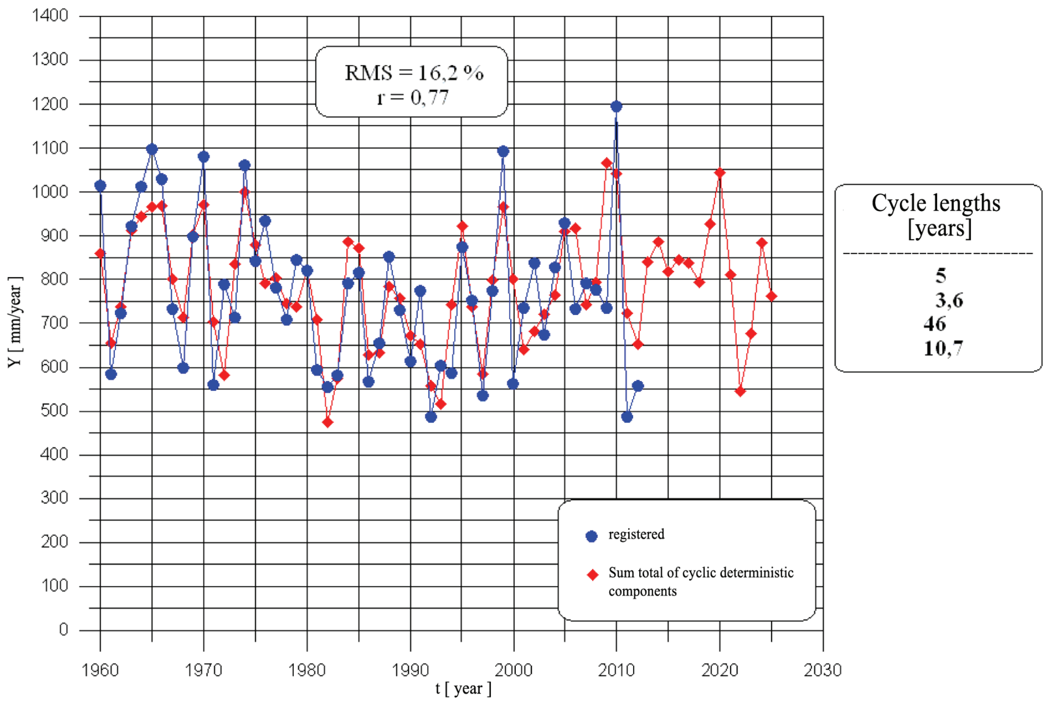

Fig. 15. Variation in the annual maximum of annual precipitation value in the Mátraalja and Bükkalja regions. (Prognosticated on the basis of four cyclic components.)

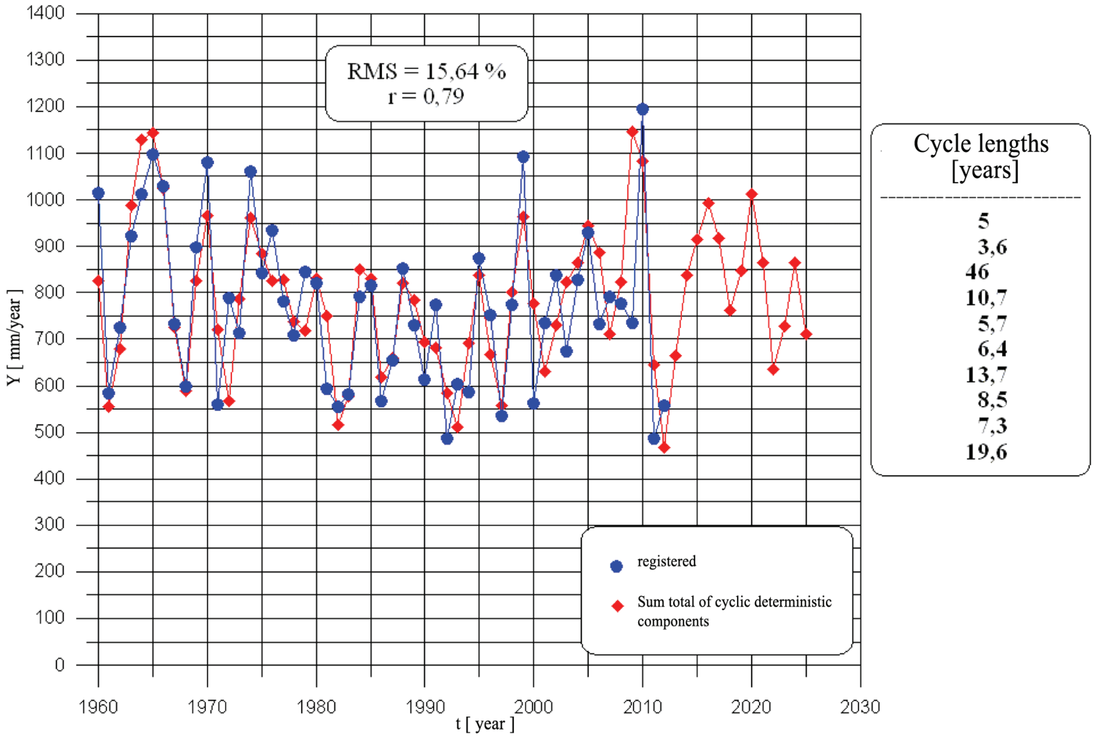

Fig. 16. Variation in the annual maximum of annual precipitation value in the Mátraalja and Bükkalja regions (Prognosticated on the basis of ten cyclic components.)

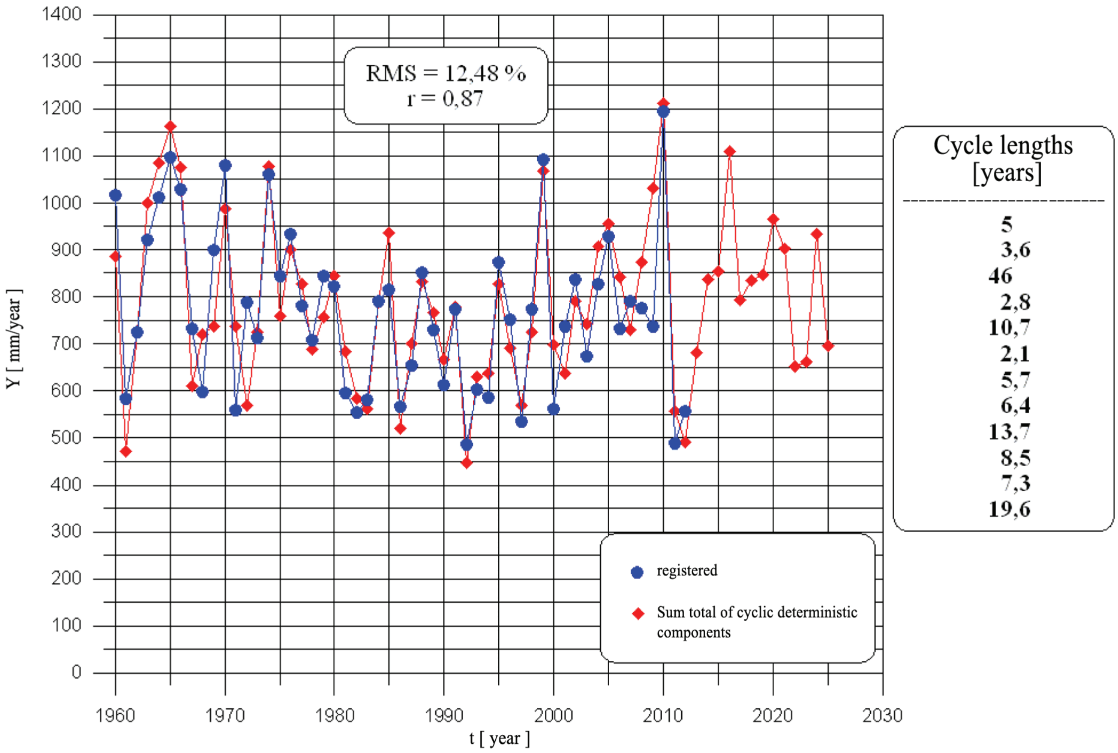

Fig. 17. Variation in the annual maximum of annual precipitation value in the Mátraalja and Bükkalja regions. (Prognosticated on the basis of twelve cyclic components.)

On the basis of the classical statistical parameters (RMS = 16.2%, r = 0.77) it can be concluded here, too, that between 1960 and 2012, the four deterministic major cycles decisively determined absolute maximums (*Fig. 15*). Taking the six minor cycles into account hardly improves classical statistical parameters (RMS = 15.6 %, r = 0.79) in this case, either, but the prognosticated sections in *Figs. 15* and *16* are significantly different here, too. Taking into account the two short cycles (2.1 years and 2.8 years), also appearing here with a high amplitude, has considerably improved classical statistical indicators (RMS = 12.5%, r = 0.87) (*Fig. 17*).

On the basis of the data in *Fig. 14*, for the purpose of practical utilisation it can be underlined in the prognosis, that the exceedingly high, 1079 mm/year amount of precipitation of 2010 – a uniquely high value in the last 53 years – will not recur in the next 12–15 years. The 850–900 mm/year annual precipitation, having occurred several times in previous years (1965, 1970, 1999) may 'probably be expected' in 2016. On the other hand, it is good news that in the coming 12–15 years, no annual precipitation below 500 mm/year, causing severe drought, may be expected.

The 1100 mm/year maximum precipitation prognosticated for 2016 (see *Fig. 17*), remains 100 mm/year below the round 1200(1195) mm/year value of 2010 but may reach the 1100 mm peak data of the years 1965, 1970, 1974, and 1999.

6. Variation in time of precipitation properties between the years 1960 and 2025

With the combined handling of the actual data for the years 1960–2012, presented in *Table 1* and *Figs. 14* and *17*, and the prognosis data in *Figs. 14* and *17* related to the Mátra+Bükk region, the time function of the variation of annual precipitation, and the absolute maximum precipitation values for the years 1960–2025 have been determined with the conventional statistical method.

The function in *Fig. 18* shows a constancy of 620–605 mm/year of annual (average) precipitation with 0.23 = 23% empirical deviation ($D_{deg}/Y_{average}$). The correlation coefficient characterizing the closeness of the function determined from the data of the 65-year time series is r^2 = 0.00048, which indicates the independence of the two variables of annual precipitation (average) and time (years) according to conventional statistical interpretation.

Fig. 19 shows the regression function determined on the basis of actual and prognosed annual absolute maximum precipitation data between the years 1960 and 2025. With an acceptable (reliable) 19% corrected empirical deviation and a r^2 = 0,00027 regression coefficient, the function predicts the constancy of the annual absolute maximum in the statistical sense while, for example, it predicts a 1100 mm precipitation maximum for 2016.

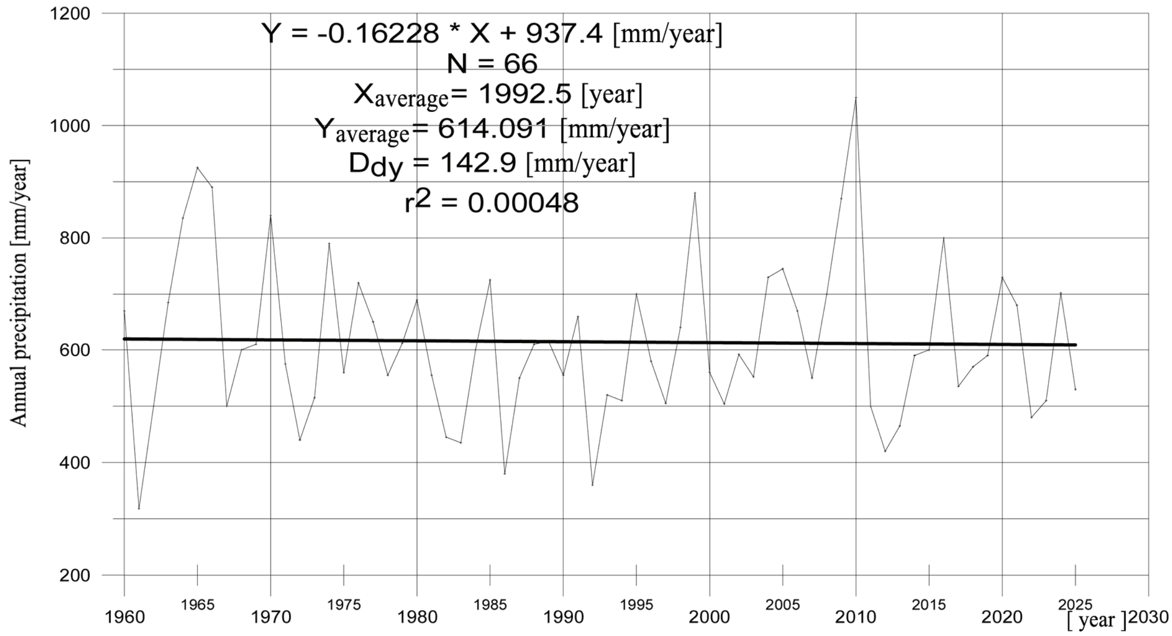

Fig. 18. Regression function of the variation in time of the annual precipitation conditions (1960-2012) and prognosis data (2013-2025) of the Mátra-Bükkalja region.

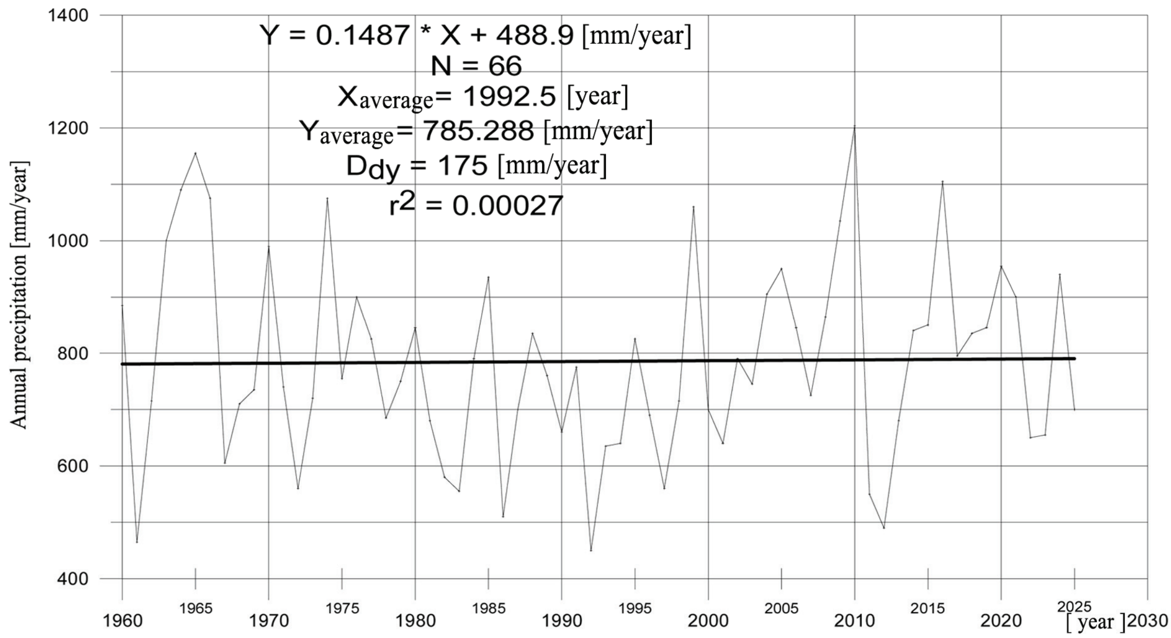

Fig. 19. Regression function of the variation in time of the annual absolute maximum precipitation properties (1960–2012) and prognosis data (2013–2025) of the Mátra-Bükkalja region.

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